Homework #7 Solutions Summer 2012

Problem 1 (a) $(3+4i)(4+3i) = 3\cdot 4 - 4\cdot 3 + i(16+9) = \boxed{25i}$

(b) $\dfrac{3+4i}{4+3i} = \dfrac{(3+4i)(4-3i)}{(4+3i)(4-3i)} = \dfrac{12+12+i(16-9)}{4^2+3^2} = \dfrac{24+7i}{25} = \boxed{\dfrac{24}{25} + \dfrac{7}{25}i}$

(c) $e^{\frac{3}{4}\pi i} = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

(d) $e^{\left(4-\frac{\pi}{3}i\right)} = e^4\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) = e^4\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) =$

$= \dfrac{e^4}{2} - i\dfrac{e^4\sqrt{3}}{2}$

Problem 2 (a) The characteristic equation is

$$r^2+4r+29=0$$

$$D = 16-4\cdot 29 = 16-116 = -100$$

$$r_{1,2} = \frac{-4\pm\sqrt{-100}}{2} = \frac{-4\pm 10i}{2} = -2\pm 5i$$

The general solution is

$$\boxed{y(t) = c_1 e^{-2t}\cos 5t + c_2 e^{-2t}\sin 5t}$$

(b) $y\left(-\frac{\pi}{2}\right) = 3 \Leftrightarrow c_1 e^{\pi}\underbrace{\cos\left(-\frac{5\pi}{2}\right)}_{=0} + c_2 e^{\pi}\underbrace{\sin\left(-\frac{5\pi}{2}\right)}_{=-1} = -c_2 e^{\pi} = 3 \Rightarrow \boxed{c_2 = -3e^{-\pi}}$

$y'(t) = -2c_1 e^{-2t}\cos 5t - 5c_1 e^{-2t}\sin 5t - 2c_2 e^{-2t}\sin 5t +$

$+5c_2 e^{-2t}\cos 5t \Rightarrow$ (since $\cos\left(-\frac{5\pi}{2}\right)=0$ and $\sin\left(-\frac{5\pi}{2}\right) = -1$)

$y'\left(-\frac{\pi}{2}\right) = 5c_1 e^{\pi} + 2c_2 e^{\pi} = 5c_1 e^{\pi} - 6e^{-\pi}e^{\pi} = 5c_1 e^{\pi} - 6 \Rightarrow$

$y'\left(-\frac{\pi}{2}\right) = -4 \Leftrightarrow \quad 5c_1 e^{\pi} - 6 = -4 \Rightarrow \quad 5c_1 e^{\pi} = 2 \Rightarrow \boxed{c_1 = \frac{2}{5}e^{-\pi}} \Rightarrow$

$$\boxed{y(t) = \frac{2}{5}e^{-\pi}e^{-2t}\cos 5t - 3e^{-\pi}e^{-2t}\sin 5t}$$

$\lim_{t\to+\infty} y(t) \longrightarrow 0$