Homework Assignment 8 in Differential Equations, MATH308-FALL 2016

due October 24, 2016 *Sections covered: section 7.5 (the case of distinct real eigenvalues), section 7.6 (the case of complex eigenvalues)*

1. Consider the following system of linear differential equations:

$$\begin{cases} x_1' &= -2x_1 + 3x_2 - x_3 \\ x_2' &= 4x_1 - x_2 + x_3 \\ x_3' &= = -4x_1 - 3x_2 - 5x_3 \end{cases} \tag{1}$$

Note that the matrix of this system is the same as in problem 3 of the previous homework assignment and we know that that the eigenvalues of it are 2, −4, −6. Use this information to solve the following:

(a) Find the general solution of the system (1).

(b) Find the solution of the the system (1) satisfying the initial condition $\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} -7 \\ 9 \\ -10 \end{pmatrix}$

2. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= -7x_1 + 5x_2 \\ x_2' &= -4x_1 + x_2 \end{cases} \tag{2}$$

(a) Find the general solution of the system (2).

(b) If $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is a solution of (2), what is the limit of $x(t)$ as $t \to +\infty$. Does this limit depend on initial conditions?

(c) Find the solution of the system (2) satisfying the initial conditions: $x_1(0) = 5, \quad x_2(0) = -4.$

3. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= -x_1 + 3x_2 - x_3 \\ x_2' &= -4x_1 + x_2 + 4x_3 \\ x_3' &= 2x_2 - x_3 \end{cases} \tag{3}$$

(a) It is known that 1 is an eigenvalue of the corresponding matrix. Find the general solution of the system (3).

(b) Find the solution of the the system (3) satisfying the initial condition $\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$

(c) Find all $\alpha_1, \alpha_2, \alpha_3$ such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$ is the solution of the system (3) with initial condition $x(0) = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$ then $x(t) \to 0$ as $t \to +\infty$.

(d) Find all $\beta_1, \beta_2, \beta_3$ such that if $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix}$ is the solution of the system (3) with initial condition $x(0) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$ then $x(t) \to 0$ as $t \to -\infty$.