# Homework Assignment 8 in Differential Equations, MATH308-SPRING 2015

due March 4, 2015

Topics covered : *the method of variation of parameter, mechanical and electrical vibrations, forced vibrations (it corresponds to sections 3.6, 3.7, and 3.8 in the textbook) ; use that the gravitational acceleration* $g = 32\frac{ft}{s^2}$

1. Use the method of variation of parameter to find the general solution of the given differential equation:
   (a) $y'' + y = \cos^2 t$;
   (b) $y'' + 3y' + 2y = \dfrac{1}{1+e^t}$
2. (a) Determine $\omega_0 > 0$, $R > 0$ and $\delta \in [0, 2\pi)$ so as to write the expression $2\cos 4t - 5\sin 4t$ in the form $R\cos(\omega_0 t - \delta)$ ; (you can use a calculator to determine an approximate value of $\delta$);
   (b) A mass weigh 8 lb stretches a spring 6 in. Assume that there is no damping. If after this the mass is pushed 2 in up and then set in motion with upward velocity of 4 in/s, determine the position $u$ of the mass at any time $t$. Find the natural frequency, the period, the amplitude, and the phase of the motion (you can use calculator to determine the phase).
   (c) Assume that in the case of the spring-mass system of item (b) there is also a damping and we can change the damping constant. What is the critical damping constant?
3. A spring is stretched 1.5 in by a mass that weighs 4 lb. The mass is attached to a dashpot mechanism that has a damping constant of $2\frac{lb\cdot s}{ft}$ and is acted on by an external force of $3\cos 4t$ lb. In both items below you can use the relevant formulas from section 3.8. Determine the steady state solution of this system;