

Homework Assignment 8 in Differential Equations, MATH308-FALL 2017
due April 18, 2017

Problems 1 and 2 are obligatory and you can solve only one of the Problems 3 or 4 to get 100: if you solve both of the last two problems you get bonus up to 33 points

Sections covered: end of 7.5 (the case when there are repeated eigenvalues and a basis of eigenvectors) & 7.8 (when there are repeated eigenvalues but no basis of eigenvectors); notes 26 and 27.

1. Consider the following system of linear differential equations:

$$\begin{cases} x'_1 &= x_1 + 2x_2 - x_3 \\ x'_2 &= 4x_1 - x_2 + 2x_3 \\ x'_3 &= 4x_1 - 4x_2 + 5x_3 \end{cases}$$

It is known that the characteristic polynomial of the matrix of the system is $-\lambda^3 + 5\lambda^2 - 3\lambda - 9$.

- (a) Find all eigenvalues of the corresponding matrix and for each eigenvalue of the matrix determine its algebraic and geometric multiplicities;
(b) Find the general solution.
2. Given the following system of linear differential equations:

$$\begin{cases} x'_1 &= -10x_1 - 27x_2 + 5x_3 \\ x'_2 &= 7x_1 + 24x_2 - 5x_3 \\ x'_3 &= -2x_1 + 18x_2 - 8x_3 \end{cases}$$

It is known that the characteristic polynomial of the matrix of the system is $-(\lambda + 3)^2(\lambda - 12)$.

- (a) Find the eigenvalues of the corresponding matrix and for each eigenvalue determine its algebraic and geometric multiplicities;
(b) Find the general solution.

For problems 3 and 4 please study notes 27.

3. Given the following system of linear differential equations:

$$\begin{cases} x'_1 &= 4x_1 + 12x_2 + 2x_3 \\ x'_2 &= -5x_1 - 12x_2 - x_3 \\ x'_3 &= -20x_1 - 24x_2 - 10x_3 \end{cases}$$

It is known that the characteristic polynomial of the matrix of the system is $-(\lambda + 6)^3$.

- (a) Find the eigenvalues of the corresponding matrix and for each eigenvalue determine its algebraic and geometric multiplicities;
(b) Find the general solution.
4. Given the following system of linear differential equations:

$$\begin{cases} x'_1 &= 2x_1 + 2x_2 + x_3 \\ x'_2 &= -\frac{3}{2}x_1 - 2x_2 - \frac{1}{2}x_3 \\ x'_3 &= -8x_1 - 8x_2 - 3x_3 \end{cases}$$

It is known that the characteristic polynomial of the matrix of the system is $-(\lambda + 1)^3$.

- (a) Find the eigenvalues of the corresponding matrix and for each eigenvalue determine its algebraic and geometric multiplicities;
(b) Find the general solution.