Homework Assignment 8 in Differential Equations, MATH308-SUMMER 2012

due June 20, 2012

Topics covered : *mechanical and electrical vibrations; linear differential equations of second order with constant coefficients: the case of repeated roots (it corresponds to sections 3.7 and 3.4 in the textbook) ; use that the graviational acceleration* $g = 32\frac{ft}{s^2}$

1. (a) Determine $\omega_0 > 0$, $R > 0$ and $\delta \in [0, 2\pi)$ so as to write the expression $-3\cos 6t + 4\sin 6t$ in the form $R\cos(\omega_0 t - \delta)$; (you can use a calculator to determine an approximate value of δ);

   (b) A mass weigh 16 lb stretches a spring 1.5 in. Assume that there is no damping. If the mass pulled an addition 4 in and then set in motion with upward velocity of 1 ft/s, determine the position u of the mass at any time t. Find the natural frequency, the period, the amplitude, and the phase of the motion (you can use calculator to determine the phase).

   (c) Assume that in the case of the spring-mass system of item (b) there is also a damping and we can change the damping constant. What is the critical damping constant?

2. Consider the differential equation $y'' - 6y' + 9y = 0$.

   (a) Find the general solution of this equation;

   (b) Find the solution of this equation satisfying the initial conditions $y(0) = \alpha$, $y'(0) = -2$;

   (c) For the solutions obtained in the previous item find the values of α , if any, for which the solutions tends to $+\infty$ as $t \to +\infty$ and the values of α, if any, for which the solutions tend to $-\infty$ as $t \to +\infty$.