Homework Assignment 9 in Differential Equations, MATH308-FALL 2016

due October 31, 2016 *Sections covered: end of 7.5 (the case when there are repeated eigenvalues and a basis of eigenvectors)& 7.8 (when there are repeated eigenvalues but no basis of eigenvectors)*

1. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= x_1 - 16x_2 \\ x_2' &= x_1 + 9x_2 \end{cases} \tag{1}$$

   (a) Find the general solution of the system (1).

   (b) If $x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ is a solution of (1), what is the limit of $x(t)$ as $t \to -\infty$. Does this limit depend on initial conditions?

   (c) Find the solution of the system (1) satisfying the initial conditions: $x_1(0) = -4, \quad x_2(0) = 5$.

2. Consider the following system of linear differential equations:

$$\begin{cases} x_1' &= -16x_1 - 3x_2 - 21x_3 \\ x_2' &= 12x_1 - x_2 + 21x_3 \\ x_3' &= 12x_1 + 3x_2 + 17x_3 \end{cases}$$

   Iis known that the characteristic polynomial of the matrix of the system is equal to $-\lambda^3 + 48\lambda + 128$ and $\lambda = -4$ is an eigenvalue of this matrix.

   (a) Find all other eigenvalues of the corresponding matrix and for each eigenvalue of the matrix (including the given eigenvalue $\lambda = -4$) determine its algebraic and geometric multiplicities;

   (b) Find the general solution.

3. Given the following system of linear differential equations:

$$\begin{cases} x_1' &= 11x_1 + 19x_2 - 14x_3 \\ x_2' &= -15x_1 - 27x_2 + 22x_3 \\ x_3' &= -12x_1 - 22x_2 + 19x_3 \end{cases}$$

   It is known that the characteristic polynomial of the matrix of the system is equal to $-\lambda^3 + 3\lambda^2 - 4$.

   (a) Find the eigenvalues of the corresponding matrix and for each eigenvalue determine its algebraic and geometric multiplicities;

   (b) Find the general solution.

4. **Bonus-**30 **points** Given the following system of linear differential equations:

$$\begin{cases} x_1' &= 6x_1 - 2x_2 \\ x_2' &= 5x_1 - x_2 - x_3 \\ x_3' &= -\frac{7}{2}x_1 + 3x_2 + 4x_3 \end{cases}$$

   (a) Find the eigenvalues of the corresponding matrix and for each eigenvalue determine its algebraic and geometric multiplicities;

   (b) Find the general solution.