

Homework Assignment 9 in Differential Equations, MATH308-Spring 2017

due April 25, 2017

Topics covered : *The Phase Plane: linear systems (section 9.1): types of critical (equilibrium) points and stability; sketch of the phase portrait for planar linear systems.*

For each of the following systems

- i) Find and *classify* the critical (equilibrium) point and determine whether it is stable, asymptotically stable, or unstable;
- ii) *Sketch* the phase portrait of the system (indicating *direction of motion* along trajectories by arrows and the direction of eigenvectors, if relevant).
- iii) (**Bonus 20 points**) In the cases when eigenvalues are complex determine the shape of ellipses and spirals based on the technique discussed in class (see Remark 2 on pages 5-6 of the notes of April 20).

1.

$$\begin{cases} x_1' &= 7x_1 - 21 \\ x_2' &= 7x_2 + 14, \end{cases}$$

2.

$$\begin{cases} x_1' &= 3x_1 + 4x_2 \\ x_2' &= -4x_1 - 3x_2 + 7, \end{cases}$$

3.

$$\begin{cases} x_1' &= 6x_1 - 2x_2 - 10 \\ x_2' &= 9x_1 - 5x_2 - 7, \end{cases}$$

4.

$$\begin{cases} x_1' &= 2x_1 + 2x_2 - 6 \\ x_2' &= -3x_1 + 7x_2 + 39, \end{cases}$$

5.

$$\begin{cases} x_1' &= -11x_1 - 2x_2 + 13 \\ x_2' &= 2x_1 - 7x_2 + 5. \end{cases}$$

6.

$$\begin{cases} x_1' &= -8x_1 + 10x_2 \\ x_2' &= -5x_1 + 2x_2. \end{cases}$$