1. Given the solution $y_{1}(t)=t^{-3}$ of the differential equation $t^{2} y^{\prime \prime}+8 t y^{\prime}+12 y=0, \quad t>0$. Use the method of reduction of order to find a second solution $y_{2}(t)$ of this equation such that $\left\{y_{1}(t), y_{2}(t)\right\}$ is a fundamental set of solutions on $t>0$.
2. (a) For each of the following equations write down the form in which a particular solution should be found according to the method of undetermined coefficients (you do not need to find the value of the undetermined coefficient/coefficients here):
i) $y^{\prime \prime}+5 y^{\prime}+6 y=3 e^{-4 t}$;
ii) $y^{\prime \prime}+5 y^{\prime}+6 y=3 e^{-4 t} \sin 3 t$;
iii) $y^{\prime \prime}+5 y^{\prime}+6 y=3 e^{-2 t}$;
iv) $y^{\prime \prime}+5 y^{\prime}+6 y=3 e^{-2 t} \cos 10 t$;
v) $y^{\prime \prime}-12 y^{\prime}+36 y=5 e^{7 t}$;
vi)) $y^{\prime \prime}-12 y^{\prime}+36 y=5 e^{6 t}-2 e^{5 t} \sin 2 t$;
vii) $y^{\prime \prime}+\omega_{0}^{2} y=\cos \omega t$ (consider separately the case $\omega^{2} \neq \omega_{0}^{2}$ and the case $\omega^{2}=\omega_{0}^{2}$ );
viii) $y^{\prime \prime}-6 y^{\prime}+25 y=e^{3 t}(\cos 4 t-2 \sin 4 t)$.
(b) Find the general solution for equation in the item (a) ii);
(c) Find the general solution for equation in the item (a) vii) (consider separately the case $\omega^{2} \neq \omega_{0}^{2}$ and the case $\omega^{2}=\omega_{0}^{2}$ ).
