

## Answers to the sample exam 1

1.  $xe^{-y} - y^2 = C$ ;
2. (a) i)  $y(t) = C_1e^{-3t} + C_2e^{-t}$ ;  
(b) 0
3. (a)  $y(t) = -\frac{1}{2}e^{2t} + \left(a + \frac{1}{2}\right)e^{10t}$ ;  
(b) If  $a > -\frac{1}{2}$ , then  $\lim_{t \rightarrow +\infty} y(t) = +\infty$ ;  
If  $a \leq -\frac{1}{2}$ , then  $\lim_{t \rightarrow +\infty} y(t) = -\infty$ ;  
 $a_0 = -\frac{1}{2}$ ;
5. (a) Equilibrium points are  $-2, 0, 2$ ;  
(c) If  $y(0) = -1$ , then  $\lim_{t \rightarrow +\infty} y(t) = 0$  and  $\lim_{t \rightarrow -\infty} y(t) = -2$ ;  
(d) If  $y(0) = 1$ , then  $\lim_{t \rightarrow +\infty} y(t) = 0$  and  $\lim_{t \rightarrow -\infty} y(t) = 2$ ;  
(e) If  $y(0) = 3$ , then  $y(t)$  is monotonically increasing and it approach  $+\infty$  (in finite time).
6.  $y^2 + 2 = Ce^{-1/x} \Leftrightarrow y = \pm\sqrt{Ce^{-1/x} - 2}$ .