MATH 308, Spring 2012
EXAM 1 - VERSION A

LAST NAME (print)______________________________ FIRST NAME : ________________________

UIN: ____________________________________SECTION #: ________________ SEAT#: ________

DIRECTIONS:

- The use of a calculator, laptop or computer is prohibited.
- In all problems present your solutions in the space provided.
- Be sure to read the instructions to each problem *carefully*.
- Use a pencil and be neat. If I can't read your answers, then I can't give you credit.
- *Show all your work* and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____

DO NOT WRITE BELOW!

page 2	page 3	page 4	page 5	page 6	page 7	page 8	page 9	Total
12	18	16	12	9	9	10	14	100

GRADE:

1. [12pts] Determine whether the differential equation $e^{-y} - (2y + xe^{-y})\dfrac{dy}{dx} = 0$ is exact. If it is exact, find the general solution.

2. Consider differential equations

$$y'' + 4y' + \alpha y = 0,$$

where α is a parameter.

(a) [15pts] In each of the following 3 cases find the general solution of the differential equation corresponding to the given α, if:

i) $\alpha = 3$

ii) $\alpha = 4$

iii) $\alpha = 13$

(b) [3pts] To what value do all solutions tend in all cases of item a) as $t \to +\infty$.

3. (a) [9pts] Solve the initial value problem

$$y' - 10y = 4e^{2t}, \quad y(0) = a \tag{1}$$

(b) [7pts] How do the solutions of (1) behave as t goes to $+\infty$? Show that this behavior depends on the choice of the initial value a and find the value a_0 for which the transition from one type of behavior to another occurs;

4. [12pts] Given the solution $y_1(t) = t^2$ of the differential equation $t^2y'' + 2ty' - 6y = 0, \quad t > 0$.
Use the method of reduction of order to find a second solution $y_2(t)$ of this equation such that $\{y_1(t), y_2(t)\}$ is a fundamental set of solutions on $t > 0$.

5. (a) [3pts] Find all equilibrium points of the differential equation:

$$y' = y^3 - 4y \tag{2}$$

(b) [6pts] Sketch the direction field for the equation (2).

-the problem is continued on the next page-

(c) [3pts] Let $y(t)$ be the solution of equation (2) satisfying the initial condition $y(0) = -1$. Based on the sketch of the direction field from item a), find the limit of $y(t)$ when $t \to +\infty$ and the limit of $y(t)$ when $t \to -\infty$ (for this you do not need to find $y(t)$ explicitly).

(d) [3pts] Let $y(t)$ be the solution of equation (2) satisfying the initial condition $y(0) = 1$. Based on the sketch of the direction field from item a), find the limit of $y(t)$ when $t \to +\infty$ and the limit of $y(t)$ when $t \to -\infty$ (for this you do not need to find $y(t)$ explicitly).

(e) [3pts] Let $y(t)$ be the solution of equation (2) with $y(0) = 3$. Based on the sketch of the direction field from item a) decide whether $y(t)$ is monotonically decreasing or increasing and find to what value it approaches when t increases (the value might be infinite).

6. [10pts] Solve the following differential equation (find the general solution): $2x^2yy' - y^2 - 2 = 0$.

7. [14pts] Using the method of undetermined coefficients, find the general solution of the differential equation

$$2y'' - 2y' + 5y = e^{3t}.$$

LAST NAME (print)____

FIRST NAME (print) ____