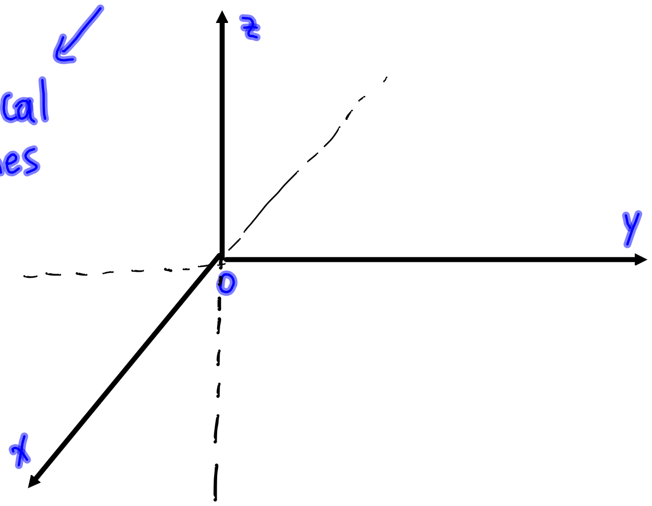
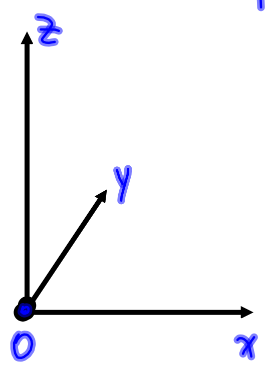
11.1: Three-dimensional Coordinate System $\mathbb{R}^3$

The three-dimensional coordinate system consists of the **origin** O and the **coordinate axes**: x-axis, y-axis, z-axis. The coordinate axes determine 3 **coordinate planes**: the xy-plane, the xz-plane and yz-plane. The coordinate planes divide space into 8 parts, called octants.

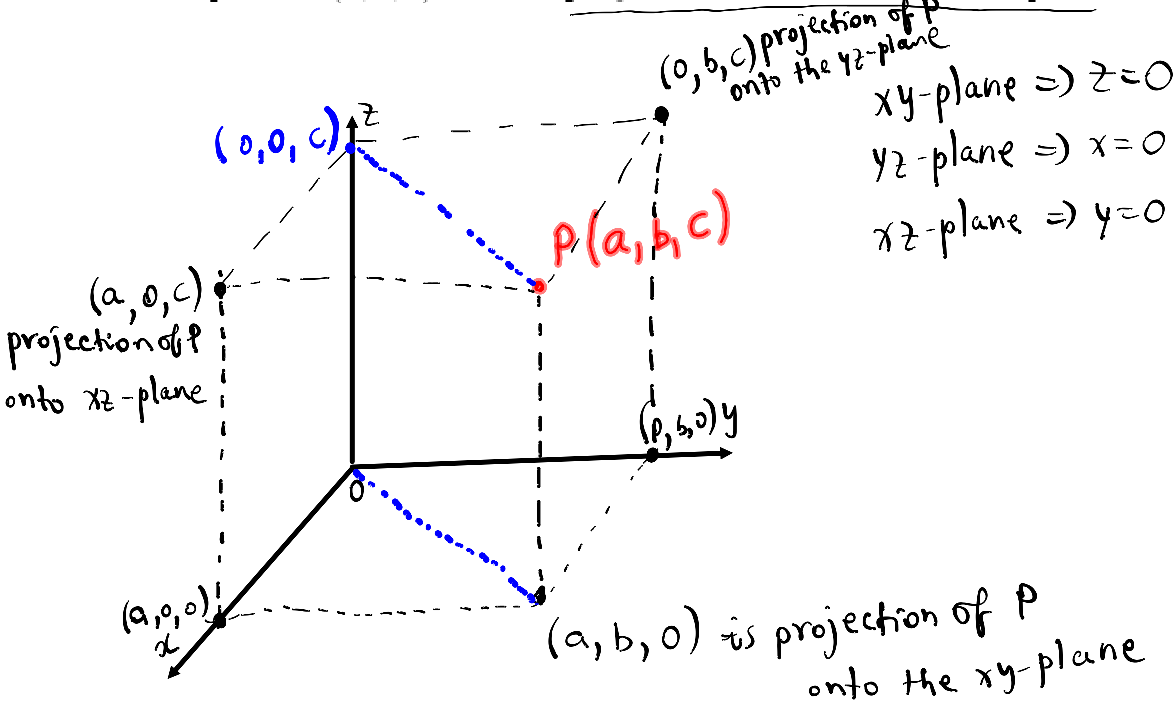
horizontal plane

vertical planes





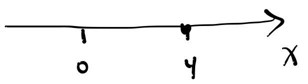
Representation of point $P(a, b, c)$ and its projections on the coordinate planes:



Example. Graph the following regions:

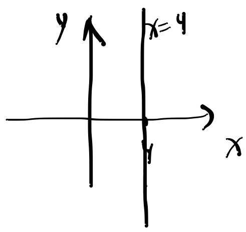
(a) $x=4$ in **R**, $\mathbf{R}^2$, $\mathbf{R}^3$

$\mathbb{R}$



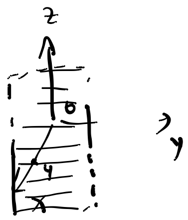
point

$\mathbb{R}^2$



line
$(4, b)$

$\mathbb{R}^3$

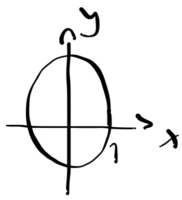


$(4, b, c)$
plane

(b) $x^2+y^2=1$ in $\mathbf{R}^2$, $\mathbf{R}^3$.

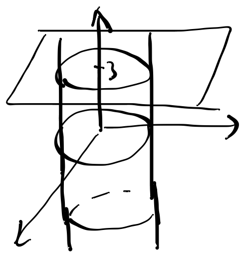
$\mathbb{R}^2$ $x^2+y^2=1$

circle
curve



$\mathbb{R}^3$ $x^2+y^2=1$, $-\infty < z < \infty$

circular
cylinder
surface

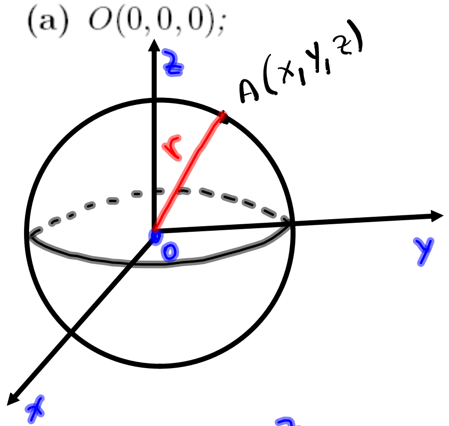


- **Distance formula in $\mathbb{R}^3$:** The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

EXAMPLE 1. *Find an equation of a sphere with radius r and center*

(a) $O(0, 0, 0)$;

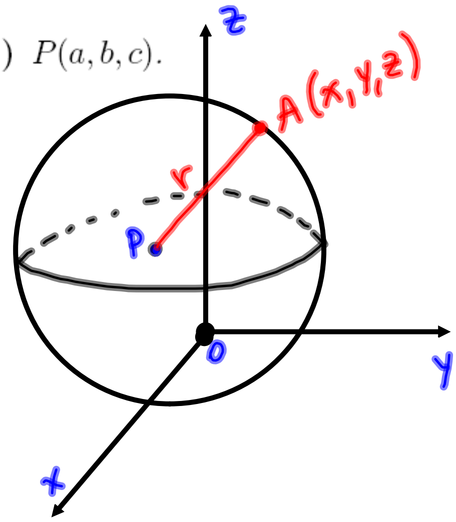


$|OA| = r$

$\sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = r$

$\boxed{x^2 + y^2 + z^2 = r^2}$

(b) $P(a, b, c)$.



$|PA| = r$

$\sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2} = r$

$\boxed{(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2}$

EXAMPLE 2. *Show that the equation $x^2 + y^2 + z^2 + x - 2y + 6z - 2 = 0$ represents a sphere, and find its center and radius.*

Comple square $\quad (a \pm b)^2 = a^2 \pm 2ab + b^2$

$$\underbrace{x^2 + \overset{2\cdot\frac{1}{2}x}{x} + \left(\frac{1}{2}\right)^2}_{\left(x+\frac{1}{2}\right)^2} - \left(\frac{1}{2}\right)^2 + \underbrace{y^2 - \overset{2\cdot 1\cdot y}{2y} + 1^2}_{(y-1)^2} - 1^2 + \underbrace{z^2 + \overset{2\cdot 3\cdot z}{6z} + 3^2}_{(z+3)^2} - 3^2 = 2$$

$$\left(x + \frac{1}{2}\right)^2 + (y-1)^2 + (z+3)^2 = 2 + \frac{1}{4} + 1 + 9 = 12 + \frac{1}{4} = \frac{49}{4}$$

We have equation of sphere centered

at $\left(-\frac{1}{2}, 1, -3\right)$ with $r = \sqrt{\frac{49}{4}} = \frac{7}{2}$