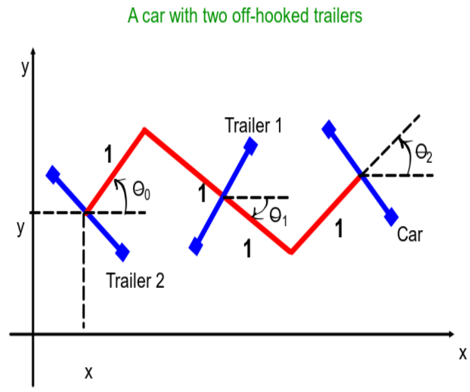
Homework Assignment 1 in Geometric Control Theory, MATH666, Fall 2013

due Sept 16, 2013

Consider the system of a car with 2 off-hooked passive trailers. A car and trailers are represented by two driving wheels connected by two axle hooked as in the figure.



Assume that the length of the axels from the mid of the car/trailers to the point of the hooking with the previous and next vehicle is equal to 1. As in the model for the car with the directly hooked discussed in class, assume that (x, y) are coordinates of the middle point of the axis of the last trailer, θ_2 is the angle of the pair of the wheel of the car with respect to the x-axis, and θ_i, $0 \le i \le 1$, are the angles of the pair of wheels of the $(n-i)$th trailer with respect to the x-axis. The state of the system is parametrized by $q = (x, y, \theta_0, \theta_1, \theta_2)$, i.e. it is $M := R^2 \times S^1 \times S^1 \times S^1 = R^2 \times \mathbb{T}^3$, where $\mathbb{T}^3$ is the 3-dimensional torus. The wheels of each body are constrained to roll without slipping, i.e. the velocity of each body is in the direction parallel to the direction of the wheels. We control the linear velocity of the car (by the control v) and the angular velocity of the car (by the control ω), where $(v, w) \in \{(\pm 1, 0), (0, \pm 1)\}$.

1. Write the control system on M corresponding to this model;

2. Find all points $q \in M$ where the corresponding control system is bracket generating, i.e. such that $Lie_q = T_qM$. Indicated the degree of nonholonomy of the system at every such q (recall that the degree of nonholonomy at q is the minimal integer $\mu(q)$ such that $Lie_q^{\mu(q)} = T_qM$.

3. Assume that $\widetilde{M}$ denotes the set of points obtained in the previous item. Prove that the control system is controllable on $\widetilde{M}$, i.e. for any $q_0 \in \mathcal{M}$ the attainable set $\mathcal{A}_{q_0}$ from the point q_0 coincides with $\widetilde{M}$.