Homework assignment 2 — Spring 2016 - MATH622

due February 18, 2016 at the beginning of class

1. Problem 23, page 52 of Warner.

2. Let $C \subset \mathbb{R}^2$ be the unit circle, and let $S \subset \mathbb{R}^2$ be the boundary of the square centered at the origin:

$$S = \Big\{(x,y) : \max\{|x|,|y|\} = 1\Big\}$$

   Show that there is a homeomorphism $F : \mathbb{R}^2 \to \mathbb{R}^2$ such that $F(C) = S$, but there is no *diffeomorphism* with the same property (Hint: Consider what F does to the tangent vector to a suitable curve in C).

3. (a) Consider $\mathbb{R}^2$ with the standard coordinates (x,y). Let $V = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$. Compute the coordinate representation of the vector field V in polar coordinates.

   (b) Given two vector fields in $\mathbb{R}^3$ (with standard coordinates (x,y,z))

$$V_1 = \frac{\partial}{\partial x}, \quad V_2 := \frac{\partial}{\partial y} + (xz + \frac{x^3}{3} + xy^2 + x^3y^2)\frac{\partial}{\partial z}.$$

   i. Calculate the following Lie brackets: $[V_1, V_2]$, $\big[V_1, [V_1, V_2]\big]$, $\big[V_2, [V_1, V_2]\big]$.

   ii. Find all points $m \in \mathbb{R}^3$ for which $[V_1, V_2](m)$ is in the linear span of $V_1(m)$ and $V_2(m)$.

   iii. Find all points $m \in \mathbb{R}^3$ for which both $\big[V_1, [V_1, V_2]\big](m)$, $\big[V_2, [V_1, V_2]\big](m)$ are in in the linear span of $V_1(m)$ and $V_2(m)$.

4. Problem 9, page 51 of Warner.

5. Problem 12, page 51 of Warner concerning details for 1.40 (b) only.

6. (**bonus question of 25 points**) Problem 16, page 51 of Warner.