Course Information

Course Number: MATH 439
Course Title: Differential Geometry of Curves and Surfaces
Section: 500
Time:
MW 05:35 pm-06:50 pm

Location: ONLINE, ZOOM,
The link to the Zoom meeting room for the class is posted in the Zoom links section of eCampus
Credit Hours: 3

Instructor Details

Instructor: **Igor Zelenko**
Office: 601J
Phone: 979-820-0620
E-Mail: zelenko@math.tamu.edu
Office Hours: **Tuesday-Friday, 11:00-12:30 pm via Zoom.**

Textbook and/or Resource Materials

1. (the main text for the first 2/3 of the course) Manfredo P. Do Carmo, *Differential Geometry of Curves and Surfaces*: Revised and updated Second Edition (Dover Books on Mathematics), the book is available electronically through the Evans library webportal

2.(optional, recommended as the main for more theoretical naterial and reference to Elements of Riemannian geometry at the very end of the course, will be replaced by my own lecture notes) Iskander A. Taimanov, *Lectures on Differential Geometry*, EMS Series of Lectures in Mathematics, 2008

3. (optional, will be replaced by my own lecture notes) Manfredo P. Do Carmo, *Differential Forms and Applications*, *Springer (Universitext)*.

4. (recommended as the main reference to Vector Calculus, but will be replaced by my own lecture notes) Jerrold E. Marsden , Anthony Tromba, *Vector Calculus*, *Sixth Edition*, *2012*.

5. Additional materials , lecture notes, lecture recordings, optional videos will be posted in e-Campus.

Course Description

This is an undergraduate introductory course in Differential Geometry.
In the first part of the course on the simplest examples of curves and surfaces in 3D we will examine several fundamental concepts such as moving frames, curvatures, covariant derivatives, parallel transport, geodesics and etc. In the second part of the course we will make

a quick introduction to manifolds, tensors, differential forms, and elements of Riemannian Geometry. In the topic of differential form the Generalized Stokes theorem, unifying Green, Stokes, and Divergence theorems you stidued in the multivariable calculus, will be discussed . We will closely follow Chapters 1-4 of Do Carmo's curves and surfaces book book (especially for examples and exercises), Part 1 and the beginning of part 2 of Taimanov's book (especially for theoretical material and elements of Riemannian Geometry),, and Do Carmo's differential forms book (for differential forms part).

Course Prerequisites

MATH 308 and 323 (or equivalent) or approval of instructor

Necessary mathematical background: Good knowledge of vector calculus (MATH221/251/253) and basic knowledge of matrices (within what is taught in Differential Equations course MATH308)

Course Learning Outcomes

You will know the main concepts of the theory of curves and surfacews, such as curvature ad torsion of a curve, the first and second fundamental forms of a surfaces, the principal , maen , and Gaussian curvature and various methods to calculate them (both in local coordinates and in local coframes using differential forms). In this way you will can answer basic question of extrinsic geometru of curves and surfaces and intrinsic geometry of surfaces.

Grading Policy

Your final grade will be determined by your performance on the homework and two exams. The grade ingredients are:

Activity	%
Exam 1 (midterm)	**20%** Take -home exam: It will be posted posted **tentatively** on Wednesday **Sept 30, 7:00 pm** with due date on next Friday **Oct 2 midnight**
Exam 2 (final exam)	**20%** Take-home exam, not-cumulative, it will be posted on Wednesday, **Nov 25** by noon with due date **Dec 1 4:30 p.m.**
Weekly Homework Assignments	**60%**: It will be posted on Wednesday with due date 0n next Wednesday, 5:30 pm
Total:	100%

Grading Scale

Range	Grade
90 -100 %	A

• **80- 89 %**	• B
• **70-79 %**	• C
• **60-69 %**	• D
• **0-59 %**	• F

Weekly Homework Assignment

Each week throughout the course there will be individual assignments whereby each student will turn in their own solutions to a give problem set. When working on the individual assignments, you may email me, discuss with classmates or look things up on the web or in a book, but you may not copy answers. You must write up your solutions in your own words, notation, and/or symbols; copying a solution from a source and referencing the source is still considered a violation of academic integrity because you are submitting work for a grade that is not your own work. If you use resources to complete your assignments, you must cite the source. For more information on plagiarism and Aggie Code of Honor, see the section on Academic Integrity below. **Weekly assignments are due on Wednesdays by 5:30 pm (CST)**.

Turning in Homework:

When turning in your assignments please follow the guidelines below:

1. On each assignment you turn in, the submitted document must have your name, the due date of the assignment, and the assignment number.
2. Save the file as LastName_Assignment#_Math666.
3. You may choose one of two ways to turn-in your assignments:
   (1) Type your solutions to the assignment in an electronic format of your choosing (Latex, Word, etc.), convert to a PDF, and then submit the PDF via eCampus.
   (2) Write your assignment on paper and then scan the paper(s) as a merged PDF document. Then submit the merged PDF document via eCampus.
4. After submitting each assignment, be sure you check the submitted document to make sure the format in which you are turning in your assignment is readable (i.e. resolution is good, scan quality is clear, etc.). If it is not easily readable, your assignment **will not be accepted**. It is the responsibility of the student to turn in work that is readable by the grader.

Note that most of the time your assignments will be graded by the math department's graduate student. If you have questions on the grading of the assignments, please let me know and I'll help you or I will get you in contact with the grader.

Late Work Policy

Late work will NOT be accepted unless you have a University approved reason and contact me within two working days of the missed assignment.

Course Schedule

WEEK	TOPIC (BELOW BY DC X.X WE DENOTE THE CORRESPONDING SECTION X.X IN DOCARMO'S BOOK # 1 IN THE LIST, BY T X.X THE CORRESPONDING SECTION X.X FROM TAIMANOV'S BOOK, BY LN MY OWN LECTURE NOTES, BY DC3 X THE CORRESPONDING CHAPTER OF DOCARMO'S BOOK #3 IN THE LIST, BY MT X.X THE CORRESPONDING SECTION FROM MARSDET & TROMBA BOOK).
WEEKS 1-2	Basic notions of theory of curves: regular curves, tangent lines, arc length, parametriztion by arc length (DC 1.2-1,3, T 1.1) .Curvature of the curve, Rigid motions. Frenet frame in R^2 and Fundamental theorem of theory of curves in R^2 (T 1.2); Frenet frame in R^3, torsion and fundamental theorem of curves in R^3(DC 1.5) Frenet Frame in R^n (brief sketch of the construction); Theory of curves under unimodular and affine transformations (optional)
WEEK 3	Review of differential calculus of vector valued map, chain rule (MT 2.3, 2.5) Implicit function theorem, inverse function theorem , regular and critical values, definiton of regular surface (DC 2.2,; T 1.4, T 2.1;MT 3.5,);
WEEK 4	Differential functions on surfaces (DC 2.3); The tangent plane; the differential of the map (DC 2.4);
WEEK 5	The first fundamental form ; Area (DC 2.5, end of T 2.1), Orientation on surfaces (DC 2.6). Gauss map; Second fundamental form, principal curvature, Gaussian and mean curvatures (DC 3.2 continued, T 2.2-2.3);
WEEK 6	The Gauss map in local coordinates; Geometric interpretation of Gaussian and mean curvature (DC 3.3); basics on minimal surfaces (DC 3.5)
WEEK 7	Isometries (DC 4.2); Christoffel symbols, the Gauss equation, and the Gauss Egregium TheoremGauss, other equations of compatibility (Codazzi equations), fundamental theorem of theory of surfaces (Bonnet theorem) (DC 4.3, T 2.1-2.5); **EXAM 1** (on the material of weeks 1-6)
WEEK 8	Covariant derivative, parallel transport; geodesics as straightest paths(DC 4.4, T 2.6). The Euler-Lagrange equation and geodesics as shortest paths (T 2.7);
WEEK 9	Gaussian curvature via the parallel transport along an infinitesimal loop (holonomy). Geodesic curvature, Gauss-Bonet theorem (DC 4.5, T 2.8) and its application such as Poincare-Hopf index theorem (DC 4.5);
WEEK 10	Introduction to smooth manifolds (T 3.1-3.2); Tensors and tensor fields (T 3.3-3.4) Introduction to differential forms (wedge product and exterior differential (DC3 Chapter 1, MT 8.5)
WEEK 11	Pull-back of forms and integration of forms on submanifolds (DC3 4.1). Generalized Stokes theorem (DC3 4.2)
WEEK 12	Extrinsic geometry of surfaces via differential forms forms (Cartan's method) (DC3 5.2); Intrinsic geometry of surfaces via differential forms (DC3 5.3)
WEEK 13-14	Elements of Riemannian Geometry: Metric tensor (T 4.1); Affine connection and covariant derivative, Chtistoffel symbols, torsion tensor; Levi-Civita Connection (T 4.3), Riemann curvature tensor (T 4.4)
WEEK 15	Review

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Attendance Policy and Make-Up Policies

- Attendance is essential to complete this course successfully.

- **Excused Absences:** University student rules concerning excused and unexcused absences, as wellas makeups, can be found at http://student-rules.tamu.edu/rule07. In particular, make-up exams and quizzes or late homework, writing assignments will NOT be allowed unless a University approved reason is given to me in writing. Notification before the absence is required when possible. Otherwise (e.g. accident, or emergency), you must notify me within two business days of the missed exam, quiz, or assignment to arrange a makeup.
- **For Fall 2020 only,** students may use the Explanatory Statement for Absence from Class form in lieu of a medical confirmation. Students must submit the Explanatory Statement for Absence from Class within two business days after the last date of absence.
- **Internet Problems:** If you experience Internet connection issues during class time, plese contact me as soon as possible to make sure you have access to content or activities you missed.

Zoom Etiquette

- **When joining class remotely via ZOOM**, please join with your audio off. When you have a question duringclass you may (1)use the "CHAT to everyone" feature to type your question, (2) use the "raise your hand" feature and wait for me to call on you, or(3) unmute yourself, politely interrupt me, and I will pause and give you time to ask your question. It is important to me that the students joining remotely are involved in the class discussion, but it is best if we do this in an organized way.
- **OFFICE HOUR ATTENDEES** When joining office hours via ZOOM, please join with your audio off. Everyone attending office hours will be joining one room, so if you would like to ask a question during office hours, please "raise your hand" and wait to be called on. If you need to speak to me privately, and have not made an individual appointment with me, please let me know through a private CHAT message and I will move you to a breakout room where we can talk one-on-one.

University policies

Attendance policies

The university views class attendance and participation as an individual student responsibility. Students are expected to attend class and to complete all assignments.

Please refer to Student Rule 7 in its entirety for information about excused absences, including definitions, and related documentation and timelines.

Makeup Work Policy

Students will be excused from attending class on the day of a graded activity or when attendance contributes to a student's grade, for the reasons stated in Student Rule 7, or other reason deemed appropriate by the instructor.

Please refer to Student Rule 7 in its entirety for information about makeup work, including definitions, and related documentation and timelines.

Absences related to Title IX of the Education Amendments of 1972 may necessitate a period of more than 30 days for make-up work, and the timeframe for make-up work should be agreed upon by the student and instructor" (Student Rule 7, Section 7.4.1).

"The instructor is under no obligation to provide an opportunity for the student to make up work missed because of an unexcused absence" (Student Rule 7, Section 7.4.2).

Students who request an excused absence are expected to uphold the Aggie Honor Code and Student Conduct Code. (See Student Rule 24.)

Academic Integrity Statement and Policy

"An Aggie does not lie, cheat or steal, or tolerate those who do."

"Texas A&M University students are responsible for authenticating all work submitted to an instructor. If asked, students must be able to produce proof that the item submitted is indeed the work of that student. Students must keep appropriate records at all times. The inability to authenticate one's work, should the instructor request it, may be sufficient grounds to initiate an academic misconduct case" (Section 20.1.2.3, Student Rule 20).

You can learn more about the Aggie Honor System Office Rules and Procedures, academic integrity, and your rights and responsibilities at aggiehonor.tamu.edu.

***NOTE**: Faculty associated with the main campus in College Station should use this Academic Integrity Statement and Policy. Faculty not on the main campus should use the appropriate language and location at their site.*

Americans with Disabilities Act (ADA) Policy

Texas A&M University is committed to providing equitable access to learning opportunities for all students. If you experience barriers to your education due to a disability or think you may have a disability, please contact Disability Resources in the Student Services Building or at (979) 845-1637 or visit disability.tamu.edu. Disabilities may include, but are not limited to attentional, learning, mental health, sensory, physical, or chronic health conditions. All students are encouraged to discuss their disability related needs with Disability Resources and their instructors as soon as possible.

***NOTE**: Faculty associated with the main campus in College Station should use this Americans with Disabilities Act Policy statement. Faculty not on the main campus should use the appropriate language and location at their site.*

Title IX and Statement on Limits to Confidentiality

Texas A&M University is committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws prohibit gender-based discrimination and sexual

harassment, including sexual assault, sexual exploitation, domestic violence, dating violence, and stalking.

With the exception of some medical and mental health providers, all university employees (including full and part-time faculty, staff, paid graduate assistants, student workers, etc.) are Mandatory Reporters and must report to the Title IX Office if the employee experiences, observes, or becomes aware of an incident that meets the following conditions (see University Rule 08.01.01.M1):

- The incident is reasonably believed to be discrimination or harassment.
- The incident is alleged to have been committed by or against a person who, at the time of the incident, was (1) a student enrolled at the University or (2) an employee of the University.

Mandatory Reporters must file a report regardless of how the information comes to their attention – including but not limited to face-to-face conversations, a written class assignment or paper, class discussion, email, text, or social media post. Although Mandatory Reporters must file a report, in most instances, you will be able to control how the report is handled, including whether or not to pursue a formal investigation. The University's goal is to make sure you are aware of the range of options available to you and to ensure access to the resources you need.

Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with Counseling and Psychological Services (CAPS).

Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University's Title IX webpage.

***NOTE**: Faculty associated with the main campus in College Station should use this Title IX and Statement on Limits of Liability. Faculty not on the main campus should use the appropriate language and location at their site.*

Statement on Mental Health and Wellness

Texas A&M University recognizes that mental health and wellness are critical factors that influence a student's academic success and overall wellbeing. Students are encouraged to engage in proper self-care by utilizing the resources and services available from Counseling & Psychological Services (CAPS). Students who need someone to talk to can call the TAMU Helpline (979-845-2700) from 4:00 p.m. to 8:00 a.m. weekdays and 24 hours on weekends. 24-hour emergency help is also available through the National Suicide Prevention Hotline (800-273-8255) or at suicidepreventionlifeline.org.

COVID-19 Temporary Amendment to Minimum Syllabus Requirements

The Faculty Senate temporarily added the following statements to the minimum syllabus requirements in Fall 2020 as part of the university's COVID-19 response.

Campus Safety Measures

To promote public safety and protect students, faculty, and staff during the coronavirus pandemic, Texas A&M University has adopted policies and practices for the Fall 2020 academic term to limit virus transmission. Students must observe the following practices while participating in face-to-face courses and course-related activities (office hours, help sessions, transitioning to and between classes, study spaces, academic services, etc.):

- Self-monitoring—Students should follow CDC recommendations for self-monitoring. **Students who have a fever or exhibit symptoms of COVID-19 should participate in class remotely and should not participate in face-to-face instruction.**
- Face Coverings—Face coverings (cloth face covering, surgical mask, etc.) must be properly worn in all non-private spaces including classrooms, teaching laboratories, common spaces such as lobbies and hallways, public study spaces, libraries, academic resource and support offices, and outdoor spaces where 6 feet of physical distancing is difficult to reliably maintain. Description of face coverings and additional guidance are provided in the Face Covering policy and Frequently Asked Questions (FAQ) available on the Provost website.
- Physical Distancing—Physical distancing must be maintained between students, instructors, and others in course and course-related activities.
- Classroom Ingress/Egress—Students must follow marked pathways for entering and exiting classrooms and other teaching spaces. Leave classrooms promptly after course activities have concluded. Do not congregate in hallways and maintain 6-foot physical distancing when waiting to enter classrooms and other instructional spaces.
- To attend a face-to-face class, students must wear a face covering (or a face shield if they have an exemption letter). If a student refuses to wear a face covering, the instructor should ask the student to leave and join the class remotely. If the student does not leave the class, the faculty member should report that student to the Student Conduct office for sanctions. Additionally, the faculty member may choose to teach that day's class remotely for all students.

Personal Illness and Quarantine

Students required to quarantine must participate in courses and course-related activities remotely and **must not attend face-to-face course activities**. Students should notify their instructors of the quarantine requirement. Students under quarantine are expected to participate in courses and complete graded work unless they have symptoms that are too severe to participate in course activities.

Students experiencing personal injury or Illness that is too severe for the student to attend class qualify for an excused absence (See Student Rule 7, Section 7.2.2.) To receive an excused absence, students must comply with the documentation and notification guidelines outlined in Student Rule 7. While Student Rule 7, Section 7.3.2.1, indicates a medical confirmation note from the student's medical provider is preferred, **for Fall 2020 only, students may use the Explanatory Statement for Absence from Class form in lieu of a medical confirmation. Students must submit the Explanatory Statement for Absence from Class within two business days after the last date of absence.**

Operational Details for Fall 2020 Courses

For additional information, please review the FAQ on Fall 2020 courses at Texas A&M University.

College and Department Policies

College and departmental units may establish their own policies and minimum syllabus requirements. As long as these policies and requirements do not contradict the university level requirements, colleges and departments can add them in this section.