

Course Information

Course Number: MATH 623
Course Title: Differential Geometry II
Section: 600
Time: TR 09:35 am-10:50 am
Location: Bloc 160
Credit Hours: 3

Instructor Details

Instructor: **Igor Zelenko**
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Office Hours: Monday noon -1 pm, Wednesday 11 am-noon via Zoom (the link to Zoom room is provided on Canvas)
Course Webpage: Canvas(<https://canvas.tamu.edu/courses/88798>)

Textbook and/or Resource Materials

Main Text:

Liviu Nicolaescu, *Lectures on Geometry of Manifolds*, 2nd Edition, World Scientific Publishing, 2007 (the 1st edition can be used as well)

Other texts (optional, all required materials will be provided via Canvas):

1. **Manfredo Perdigao do Carmo (Author)** (Francis Flaherty (Translator)), *Riemannian Geometry*, 1st Edition, Burkhauser Boston, 1992 (mainly for homework assignments and as a reference for Variational theory of geodesics and comparison theorems, mainly chapters 5, 9, 10, 11 and also some of chapter 6 there)
2. **Jurgen Jost**, *Riemannian Geometry and Geometric analysis*, 7th edition (mainly as additional reference to material discussed in class)
3. **Michael Spivak**, *A Comprehensive Introduction to Differential Geometry: Volume II* (mainly as a reference for the topic of connections on vectors and principle bundles, Chapters 6-8 there)

4. **R.W. Sharpe**, *Differential Geometry: Cartan's Generalization of Klein's Erlangen Program*, Springer Graduate Texts in Mathematics, Vol. 166 (mainly as a reference for Cartan method of equivalence in application to extrinsic geometry of submanifolds of homogeneous spaces, Chapter 3 there)
5. **S. Sternberg**, *Lectures on Differential Geometry*, AMS Chelsea Publishing, 2nd Edition (mainly as a reference to the theory of connections on principal bundles and G -structures, chapter VII there)

The material that not included on the texts above will be posted in the form of the lecture notes.

Additional references that you may like to consult include:

6. Chern, S. S., Chen, W. H., and Lam, K. S., *Lectures on differential geometry*. [Series on University Mathematics, 1](#). World Scientific Publishing Co., Inc., River Edge, NJ, 1999x+356 pp.
7. **S. Kobayashi, K. Nomizu**, *Foundations of Differential Geometry, Vol 1 (1963) and 2 (1969)*, Interscience publisher.
8. **S. Gallot, D. Hulin, J. Lafontaine**, *Riemannian Geometry*, **any edition (there are 3 editions)** (especially regarding algebraic decomposition of Riemannian curvature tensor coming from the representation theory and Weyl conformal tensor, also a great source for various comparison theorems)

Course Prerequisites

Some basic concepts from Linear algebra, Topology, and Analysis on manifolds.

The course Differential Geometry I, M622, is strongly recommended but not required. The undergraduate prerequisites are standard Calculus courses (MATH 171/151, 172/152, 221/251 or equivalent), Differential Equations (MATH 308), and Linear algebra (MATH 304 or 323). I will give all mathematical background beyond the above courses in the class.

Course Description

This is the second semester of a year-long graduate course in differential geometry. We will discuss the following topics:

1. Connections on vector bundles, the curvature tensor of a connection, the connection and curvature form of a connection in local trivialization, parallel transport, holonomy,

Ambrose -Singer theorem (formulation and discussion), geodesics. (Differential) Bianchi identity.

2. Connections on tangent bundle, its torsion tensor and form, the algebraic Bianchi identity, Riemannian manifolds and Levi-Civita connection. Algebraic properties of Riemannian curvature tensor, Ricci and scalar curvature, sectional curvature and its geometric interpretation, Weyl conformal tensor. Brief discussion of spin structures and the Young-Mills functional.
3. Basics on Lie groups and Lie algebra: a notion of Lie groups and its Lie algebra, exponential map, a construction of Lie group by its Lie algebra (brief), Lie subgroup, adjoint representations, Killing form, bi-invariant metrics on Lie groups and their Riemannian curvature tensor, elements of homogeneous spaces and symmetric spaces; Maurer-Cartan form on a Lie group.
4. Geometry of submanifolds in homogeneous spaces as an important application of the Frobenius theorem via the Maurer-Cartan form on a Lie group, the notion of the Darboux derivative of a map from a manifold to a Lie group and the fundamental theorem for Darboux derivatives via the Cartan's graph method. The Geometry of submanifolds of Riemannian manifolds: the second fundamental form, the Gauss, Ricci, and Codazzi equations, the fundamental theorem of submanifolds in R^n .
5. Elements of calculus on Riemannian manifolds: Hodge star operator, gradient, divergence, Laplacian of differential forms and harmonic forms, basic on De Rham cohomology and Hodge theorem (formulation and discussions only).
6. Variational theory of Riemannian geodesics: the energy functional and its first variation, geodesics as critical points, the second variation along geodesics, Jacobi fields, conjugate points, the Morse index theorem (formulation and discussions, the proof is a topic for final presentation together with applications in homotopy theory via Morse theory); expansion of volumes and geometric interpretation of Ricci and scalar curvature. Bonnet-Myers theorem and its consequences for the topology of a manifold. Other types of

comparison theorems like the Rauch and the volume comparison theorem are usually left for final presentations,

7. Elements of the theory of principle connections: definition, curvature form and its property.
8. Overview of Chern-Weil theory of characteristic classes: invariant polynomials, Chern and Pontryagin classes, the Euler class, the Gauss-Bonnet-Chern theorem.
9. If time permits I also discuss G-structures and their prolongations, elements of Cartan connection, with applications to Riemannian and conformal geometry, as this topic is more close to my own research.

Course Learning Outcomes

You will be able to conduct research involving various geometric objects such as moving frames , curvature tensors etc in various directions (geometric, analytic, topological, and algebraic)

Grading Policy

Your final grade will be determined by your performance on the homework and the final presentation. The grade ingredients are:

• Activity	• %
• Bi-weekly Homework Assignments	• 70%: It will be usually posted on Wednesdays with due date in 2 weeks. Homework submission will be via Canvas
• Final presentation	• 30%: In the beginning of November it will be assigned to you in advanced on a topic which continues some topic discussed in a class or on a new topic which was not covered in class. The presentation will be in the final exam period , i.e. Dec 8-Dec 15. The schedule will be determined later.
• Total:	• 100%

Grading Scale

• Range	• Grade
• 90 -100 %	• A
• 80- 89 %	• B
• 70-79 %	• C
• 60-69 %	• D
• 0-59 %	• F

TECHNOLOGY: You will need to scan and upload some of your written work as a PDF (this can be achieved with a cell phone or other technology). Course notes and other materials will be posted on Canvas.

If the university moves to on-line instruction, then exams will be proctored on-line over Zoom. To do this, the following technical requirements are needed:

- Appropriate hardware (laptop or desktop computer, a second device such as a mobile phone, high-speed internet connection)
- Appropriate software (PDF reader, Zoom on phone and computer, the latest update on an internet browser-Chrome or Firefox are recommended)

Grade complaints:

If you think a homework was graded incorrectly you have one week from the time the graded assignment was returned to you to bring the issue to the instructor's attention. No complaints after that time will be considered.

Exams: There will not be exams in this class.

Late Work Policy

Late work will NOT be accepted unless you have a University approved reason and contact me within two working days of the missed assignment.

Appeal Policy

Students have one week upon the return of individual grades to notify the instructor of any inaccuracies in their graded work. Students should bring all grade disputes to their instructor in an individual Zoom meeting. Due to FERPA privacy issues, grade disputes will not be discussed over email or in the classroom.

Working with Friends

Working together on homework is fine and encouraged, but each of you **must write up your own solutions in your own words, notation and/or symbols and write the names of your collaborators at the top left corner of your homework**. Copying a solution from a source and referencing the source is still considered a violation of academic integrity because you are submitting work for a grade that is not your own work. It is NOT permissible to discuss any aspect of any quiz, test or examination until ALL students have completed it. The penalties for violating this policy will range from an F on an assignment or test, to failing in the course.

Attendance

Attendance is essential to complete this course successfully.

Excused Absences: University student rules concerning excused and unexcused absences, as well as makeups, can be found at <http://student-rules.tamu.edu/rule07>. Make-up exams and quizzes or late

homework, writing assignments will NOT be allowed unless a University approved reason is given to me in writing. Notification before the absence is required when possible. Otherwise (e.g., accident, or emergency), you must notify me within two business days of the missed exam, quiz, or assignment to arrange a makeup.

Other Important Dates: September 3 (last day to add or drop a course), Nov 24 (Reading Day), November 25&26 (no class-- Thanksgiving), November 19 (Q-drop).

Zoom Etiquette

OFFICE HOUR ATTENDEES

When joining office hours via ZOOM, please join with your audio off. Everyone attending office hours will be joining one room, so if you would like to ask a question during office hours, please "raise your hand" and wait to be called on. If you need to speak to me privately, and have not made an individual appointment with me, please let me know through a private CHAT message and I will move you to a breakout room where we can talk one-on-one.

Class Announcements

Class announcements will be posted in [Canvas](#) and sent to your university e-mail account (Make sure to check your notification preferences to control how the course updates are sent.) It is your responsibility to check your account and the course page and get familiar with the announcements.

University Policies

Attendance Policy

The university views class attendance and participation as an individual student responsibility. Students are expected to attend class and to complete all assignments.

Please refer to Student Rule 7 in its entirety for information about excused absences, including definitions, and related documentation and timelines.

Makeup Work Policy

Students will be excused from attending class on the day of a graded activity or when attendance contributes to a student's grade, for the reasons stated in Student Rule 7, or other reason deemed appropriate by the instructor.

Please refer to Student Rule 7 in its entirety for information about makeup work, including definitions, and related documentation and timelines.

Absences related to Title IX of the Education Amendments of 1972 may necessitate a period of more than 30 days for make-up work, and the timeframe for make-up work should be agreed upon by the student and instructor” (Student Rule 7, Section 7.4.1).

“The instructor is under no obligation to provide an opportunity for the student to make up work missed because of an unexcused absence” (Student Rule 7, Section 7.4.2).

Students who request an excused absence are expected to uphold the Aggie Honor Code and Student Conduct Code. (See Student Rule 24.)

Academic Integrity Statement and Policy

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

“Texas A&M University students are responsible for authenticating all work submitted to an instructor. If asked, students must be able to produce proof that the item submitted is indeed the work of that student. Students must keep appropriate records at all times. The inability to authenticate one’s work, should the instructor request it, may be sufficient grounds to initiate an academic misconduct case” (Section 20.1.2.3, Student Rule 20).

You can learn more about the Aggie Honor System Office Rules and Procedures, academic integrity, and your rights and responsibilities at aggiehonor.tamu.edu.

Americans with Disabilities Act (ADA) Policy

Texas A&M University is committed to providing equitable access to learning opportunities for all students. If you experience barriers to your education due to a disability or think you may have a disability, please contact Disability Resources in the Student Services Building or at (979) 845-1637 or visit disability.tamu.edu. Disabilities may include, but are not limited to attentional, learning, mental health, sensory, physical, or chronic health conditions. All students are encouraged to discuss their disability related needs with Disability Resources and their instructors as soon as possible.

Title IX and Statement on Limits to Confidentiality

Texas A&M University is committed to fostering a learning environment that is safe and productive for all. University policies and federal and state laws prohibit gender-based discrimination and sexual harassment, including sexual assault, sexual exploitation, domestic violence, dating violence, and stalking.

With the exception of some medical and mental health providers, all university employees (including full and part-time faculty, staff, paid graduate assistants, student workers, etc.) are Mandatory Reporters and must report to the Title IX Office if the employee experiences, observes, or becomes aware of an incident that meets the following conditions (see University Rule 08.01.01.M1):

- The incident is reasonably believed to be discrimination or harassment.
- The incident is alleged to have been committed by or against a person who, at the time of the incident, was (1) a student enrolled at the University or (2) an employee of the University.

Mandatory Reporters must file a report regardless of how the information comes to their attention – including but not limited to face-to-face conversations, a written class assignment or paper, class discussion, email, text, or social media post. Although Mandatory Reporters must file a report, in most instances, you will be able to control how the report is handled, including whether or not to pursue a formal investigation. The University’s goal is to make sure you are aware of the range of options available to you and to ensure access to the resources you need.

Students wishing to discuss concerns in a confidential setting are encouraged to make an appointment with Counseling and Psychological Services (CAPS).

Students can learn more about filing a report, accessing supportive resources, and navigating the Title IX investigation and resolution process on the University’s Title IX webpage.

Statement on Mental Health and Wellness

Texas A&M University recognizes that mental health and wellness are critical factors that influence a student’s academic success and overall wellbeing. Students are encouraged to engage in proper self-care by utilizing the resources and services available from Counseling & Psychological Services (CAPS). Students who need someone to talk to can call the TAMU Helpline (979-845-2700) from 4:00 p.m. to 8:00 a.m. weekdays and 24 hours on weekends. 24-hour emergency help is also available through the National Suicide Prevention Hotline (800-273-8255) or at suicidepreventionlifeline.org.

Classroom Facial Mask

To help protect Aggieland and stop the spread of COVID-19, Texas A&M University urges students to be vaccinated and to wear masks in classrooms and all other academic facilities on campus, including labs. Doing so exemplifies the Aggie Core Values of respect, leadership, integrity, and selfless service by putting community concerns above individual preferences. COVID-19 vaccines and masking — regardless of vaccination status — have been shown to be safe and effective at reducing spread to others, infection, hospitalization, and death.