

# Chapter 2 Exponentials and Logarithms

The exponential function is one of the most important functions in the field of mathematics. It is widely used in a variety of applications such as compounded interest, population growth, and carbon dating. This chapter is a brief introduction to the exponential function and its inverse function, the logarithmic function. We will look at various properties of these two functions, learn to solve equations involving them, and then finally use them to model real world situations.

## 2.1 Exponential Functions

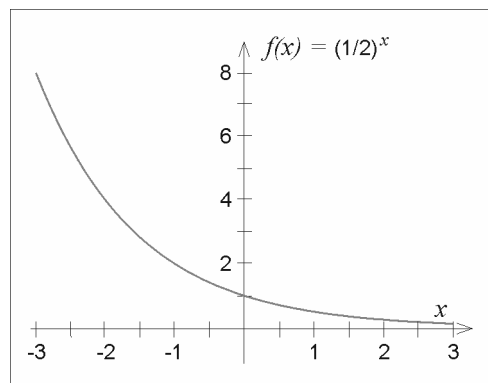
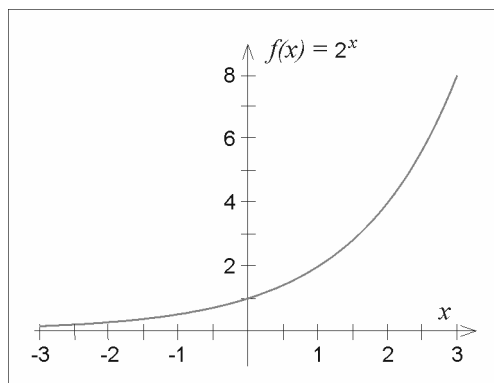
An exponential function is a function of the form  $f(x) = b^x$  where  $b > 0$  and  $b \neq 1$ . Some properties of exponential functions are summarized in Table 2.1. Figure 2.1 shows the two different types of curves that are generated by an exponential function.

Table 2.1 Properties of Exponential Functions

Let  $f(x) = b^x$  where  $b > 0$  and  $b \neq 1$ .

1. If  $b > 1$  then  $f(x)$  is an exponential growth function.
2. If  $0 < b < 1$  then  $f(x)$  is an exponential decay function.
3. The domain of  $f(x)$  is  $(-\infty, \infty)$ .
4. The range of  $f(x)$  is  $(0, \infty)$ .
5. The y-intercept is  $(0, 1)$ .

Figure 2.1 Graphs of exponential growth and decay functions.





**EXAMPLE 2.1** Determine whether each of the following are exponential growth or decay functions. Then graph each function and verify the domain, range and  $y$ -intercept of the function are as stated in Table 2.1.

a.  $f(x) = \left(\frac{3}{2}\right)^x$

b.  $f(x) = \left(\frac{1}{4}\right)^x$

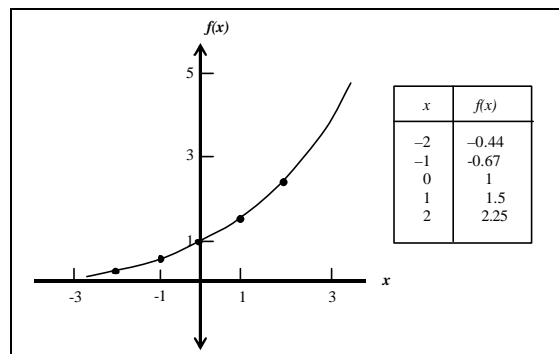


**Solution**

a. This is an exponential growth function because  $b = 1.5$  and that is greater than 1.



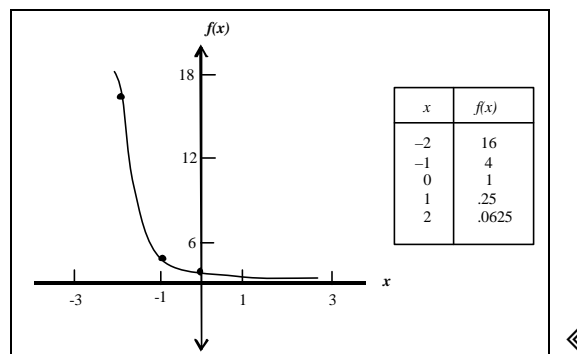
**Figure 2.2** Graph of Example 2.1 a. By inspection of the graph in Figure 2.2, the domain is  $(-\infty, \infty)$  and the range is  $(0, \infty)$ .



b. This is an exponential decay function because  $b = 0.25$  and that is less than 1.




**Figure 2.3** Graph of Example 2.1 b. By inspection of the graph in Figure 2.3, the domain is  $(-\infty, \infty)$  and the range is  $(0, \infty)$ .




The function  $f(x) = 2^x$  is an exponential function because the independent variable,  $x$ , is part of the exponent. If we were given a table of values, or a data set, how could we determine if it modeled an exponential function? One method is to calculate the ratio of successive  $f(x)$  values when the  $x$  values


are equally spaced. If the ratios are all the same (or close to the same) the table of values would model an exponential function. The value of the successive ratios is the base of the exponential function.

 **EXAMPLE 2.2** Determine if the table of values given below represents an exponential function. If so, find the base of the function.

$x$	2	3	4	5	6
$f(x)$	6.25	15.625	39.063	97.656	244.14

 **Solution** Compute the successive ratios, as shown below.


$$\frac{f(3)}{f(2)} = \frac{15.625}{6.25} = 2.5, \quad \frac{f(4)}{f(3)} = \frac{39.063}{15.625} \approx 2.5, \quad \frac{f(5)}{f(4)} = \frac{97.656}{39.063} \approx 2.5, \quad \frac{f(6)}{f(5)} = \frac{244.14}{97.656} = 2.5$$

Since all ratios are approximately equal to 2.5 we conclude that the table of values does represent an exponential growth function with a base of 2.5. 

Exponential functions have a variety of applications in the business world and it is often necessary to solve these equations for different variables. In order to solve these equations we must know the laws and definitions associated with exponential functions that are listed in Table 2.2.

 **Table 2.2** Properties and Laws of exponents  $a, b > 0$ , positive??

Let $a$ and $b$ be positive numbers and let $m$ and $n$ be real numbers. Then,	
1. $a^0 = 1$	2. $a^{-m} = \frac{1}{a^m}$ , where $a \neq 0$
3. $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	4. $a^m \cdot a^n = a^{m+n}$
5. $\frac{a^m}{b^n} = a^{m-n}$	6. $(ab)^m = a^m b^m$
7. $(a^m)^n = a^{m \cdot n}$	8. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

 **Example 2.3** Use the laws and definitions of exponents from Table 2.2 to solve the following equations for  $x$ .

a.  $5^{x+3} = 5^{2x}$

b.  $8^x = 64^{2x-5}$

c.  $3^{x+1} - \left(\frac{1}{27}\right)^{2x} = 0$

d.  $x \cdot 9^{-x} = \left(\frac{x}{3^x}\right)^2$

e.  $9^x - 12 \cdot 3^x = -27$

 **Solution**

a. The expressions on both sides of the equation have a base of 5. Since these expressions and the bases are equal we can conclude that their exponents must also be equal. So we set up an equation stating this fact and solve it for  $x$ .

$$\begin{aligned} x + 3 &= 2x \\ 3 &= x \end{aligned}$$

b. Here the bases of each term are not equal to each other. Thus, our first step is to search for a common base. We know that  $64 = 8^2$  so we can rewrite 64 as  $8^2$ . Use the laws from Table 2.2 to find

$$8^x = (8^2)^{2x-5} = 8^{2(2x-5)} = 8^{4x-10}$$

Now solve for  $x$  by setting the exponents equal,

$$\begin{aligned} x &= 4x - 10 \\ 10 &= 3x \\ \frac{10}{3} &= x \end{aligned}$$

c. Since the bases on each term are different we must rewrite 27 as  $3^3$  and apply laws and definitions of exponents,

$$3^{x+1} - \left(\frac{1}{3^3}\right)^{2x} = 3^{x+1} - (3^{-3})^{2x} = 3^{x+1} - 3^{-6x} = 0$$

Now solve for  $x$ ,

$$\begin{aligned} 3^{x+1} &= 3^{-6x} \\ x + 1 &= -6x \\ 7x &= -1 \\ x &= -\frac{1}{7} \end{aligned}$$

d. This problem differs from the previous three because there is an  $x$  in each term that accompanies the exponential function. Our strategy in solving this problem will still involve getting the same bases, but now we must factor the expression and solve using zero product property<sup>1</sup>. Begin with the left side of the equation and writing 9 as  $3^2$ ,

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<sup>1</sup> That is, in a series of products, if any one of the terms are zero, then the entire product is zero.

$$x \cdot 9^{-x} = x(3^2)^{-x} = x \cdot 3^{-2x}$$

Now work with the right side of the equation,

$$\left(\frac{x}{3^x}\right)^2 = \frac{x^2}{(3^x)^2} = \frac{x^2}{3^{2x}} = x^2 3^{-2x}$$

Next set them equal to each other,

$$x3^{-2x} = x^2 3^{-2x}$$

Rearrange to have all the variables on the left and factor.

$$\begin{aligned}x3^{-2x} - x^2 3^{-2x} &= 0 \\x3^{-2x}(1-x) &= 0\end{aligned}$$

Now by applying the zero product property we obtain:

$$x = 0, 3^{-2x} = 0, \text{ and } 1 - x = 0.$$

Since an exponential function can never equal zero,  $3^{-2x} \neq 0$ , the only solutions are  $x = 0, 1$ .

e. Since each exponential function is accompanied by an  $x$ , we will use the strategy from the previous problem. That is, we will get the bases the same and then factor.

$$(3^x)^2 - 12 \cdot 3^x = -27$$

Let  $u = 3^x$  for ease in factoring and find

$$(3^x)^2 - 12 \cdot 3^x + 27 = u^2 - 12u + 27 = (u-9)(u-3) = 0$$

The first term has the solution  $u = 9$  or  $3^x = 9 = 3^2$ , so  $x = 2$ . The second term has the solution  $u = 3$  or  $3^x = 3 = 3^1$ , so  $x = 1$ . Thus, the two solutions to the given equation are  $x = 1, 2$ .

A check of all these answers through substitution of the solution value into the original equation is encouraged. ◇

## 2.2 Applications of Exponential Functions

One of the most common types of exponential functions in the banking industry is compound interest, which can be easily derived from the simple interest formula.

**Simple Interest:** If  $P$  dollars is deposited into an account that earns an annual interest of  $r\%$  (expressed as a decimal), then the amount of interest accumulated,  $I$ , after  $t$  years is given by

$$I = Prt$$

Suppose you accumulated \$1,000 in cash from your high school graduation. If you deposit this money into an account that earns 5% simple interest, then at the end of 4 years you would have earned

$$I = (1,000)(0.05)(4) = \$200.$$

So you would have  $\$1,000 + \$200 = \$1,200$  in the account at the end of 4 years. In general, the amount in an account that earns simple interest for  $t$  years is

$$A = P + I$$

$$A = P + Prt$$

$$A = P(1 + rt)$$

Another type of interest commonly used in the banking industry is compound interest. This type of interest is often computed more than once a year and the account earns interest on the interest computed the previous compounding period. This is what makes compound interest more appealing to investors than simple interest. For example, if you take the \$1,000 accumulated from your high school graduation and deposit it into an account that earns 5% compounded quarterly, we can compute the amount in the account at the end of 1 year. To solve this we use  $A = P(1 + rt)$  with  $t = \frac{1}{4}$  because the interest is figured quarterly ( $\frac{1}{4}$  of a year).

Quarter	$A = P(1 + rt)$	Amount at the end of the Quarter
1st	$A = 1,000(1 + 0.05(\frac{1}{4})) = 1,000(1.0125)$	\$1,012.50
2nd	$A = 1,012.5(1.0125) = 1,000(1.0125)(1.0125) = 1,000(1.0125)^2$	\$1,025.16
3rd	$A = 1,025.16(1.0125) = 1,000(1.0125)^2(1.0125) = 1,000(1.0125)^3$	\$1,037.97
4th	$A = 1,037.97(1.0125) = 1,000(1.0125)^3(1.0125) = 1,000(1.0125)^4$	\$1,050.95

If we look at the expression for  $A$ , near the end of the formula we can see a pattern that is developing for compound interest. The exponent on 1.0125 is the same as the compounding period. Thus for the 16th compounding period (4 years) the amount in the account would be  $1,000(1.0125)^{16} = 1,219.89$ . If we compare this number to the amount in the account that earned simple interest for 4 years, we see that the compounded account earned \$19.89 more. This difference is greater for larger initial deposits and when the money is left in the accounts for longer time periods.

It is without a doubt that the computation above was long and tedious. We would not want to use this method of computing for larger numbers, therefore we generalize the computation and derive a formula for compound interest.

**Compound Interest Formula:** If  $P$  dollars is deposited into an account earning  $r\%$  annual interest (expressed as a decimal) and is compounded  $n$  times a year, then the amount in the account,  $A$ , after  $t$  years is given by

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

Interest can be compounded a number of different ways. Below are the  $n$  values for the different compounding periods.

**Table 2.3** Compounding Periods

	Frequency, $n$
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

**Example 2.4** Find the amount in an account after 10 years if \$2,500 is compounded monthly at 8%.

**Solution** Here  $P = 2,500$ ,  $t = 10$ ,  $n = 12$ ,  $r = 0.08$ . Thus, the amount,  $A$ , would be

$$A = 2,500 \left( 1 + \frac{0.08}{12} \right)^{(12)(10)} \approx \$5,549.10$$

Or, we can use the compound interest calculator applet to find the same result as shown in Figure 2.4.

**Figure 2.3** Compound Interest Applet for Example 2.4

The applet interface includes the following fields and controls:

- Interest Rate %: 8.0
- Num: 120
- Payment Amount \$: 0.0
- per year: 12
- Present Value \$: 2500.0
- Final Value \$: 5549.1
- Buttons: Calculate, Final Value (dropdown), Reset

**Example 2.5** How much money should be deposited into an account that earns 6.5% compounded semi-annually if, after 7 years, the account must be worth \$10,000?

**Solution** Here  $A=10,000$ ,  $r=0.065$ ,  $n=2$ ,  $t=7$ . Plugging in these values into the formula, we have

$$10,000 = P \left( 1 + \frac{0.065}{2} \right)^{(2)(7)} = P(1.0325)^{14}.$$

Now solve for P,

$$P = \frac{10,000}{(1.0325)^{14}} \approx \$6,390.56.$$

**Figure 2.4** Compound Interest Applet for Example 2.5

Suppose you invest \$1 in an account that earns 100% interest for 1 year. How would the amount of money at the end of the year change if the interest was compounded more and more often? The compound interest formula with these values would be

$$A = 1 \left( 1 + \frac{1}{n} \right)^n$$

In Table 2.4 we see how more frequent compounding (larger  $n$ ) gives you a larger and larger  $A$ , up to a point.

**Table 2.4** Amount of Money vs. Compounding Frequency


$n$	1	10	100	1,000	10,000	100,000	1,000,000
$A$	2	2.5937	2.7048	2.7169	2.7181	2.7182	2.7182


We see that as  $n$  grows larger,  $A$  becomes approximately 2.7182. This number (called  $e$ ) is frequently used in the field of mathematics and is used in the formula for continuously compounded interest.



**Continuously Compounded Interest:** If  $P$  dollars is deposited into an account earning  $r\%$  annual interest (expressed as a decimal) and is compounded continuously, then the amount in the account,  $A$ , after  $t$  years is

$$A = Pe^{rt}$$

 **Example 2.6** Find the amount in an account after 10 years if \$2,500 is deposited into an account that earns 8% compounded continuously.


 **Solution** Here  $P = 2,500$ ,  $r = 0.08$ , and  $t = 10$ . When we substitute into the continuous compound interest formula we get:

$$A = 2,500e^{(0.08)(10)} \approx 5,563.85$$

In the module there is a continuous compounding calculator that will give the same result, shown below,

 **Figure 2.5** Continuous Compounding Applet for Example 2.6

Initial Amount, P	Number of Years, t	Interest Rate, r	Final Amount, A
2500	10	0.08	5563.85
<input type="button" value="Compute"/>		<input type="button" value="Clear Fields"/>	

So there will be approximately \$5,563.85 in the account after 10 years. Comparing this answer to the one obtained in Example 2.4 shows that continuously compounded interest earns \$14.75 more than compounding quarterly. 


## 2.3 Logarithmic Functions

We can solve exponential equations when the bases are the same, but how would we solve an equation like  $2^x = 10$ ? To solve this we need to use the inverse of an exponential function, the logarithmic function.

**Definition of Logarithm:** For all  $b > 0$ ,  $b \neq 1$ , and  $y > 0$ ,  $y = b^x$  if and only if  $\log_b y = x$

Note:  $\log_b y = x$  is read “log to the base b of y equals x”.

Two commonly used logarithmic functions are the common logarithm ( $\log_{10}$ ) and the natural logarithm ( $\log_e$ ). Typically we write log for  $\log_{10}$  and ln for  $\log_e$ .

 **Example 2.7** Rewrite  $6^2 = 36$  and  $e^k = 4$  as logarithmic equations.

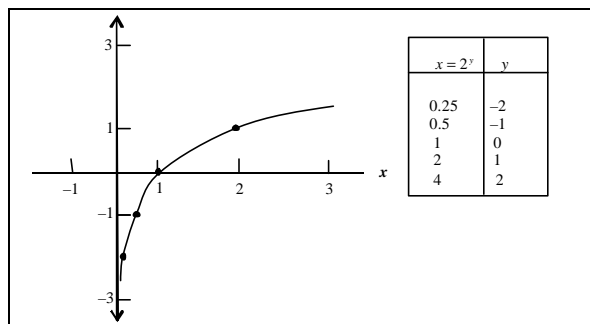
✓ **Solution**  $6^2 = 36$  is in the exponential form,  $y = b^x$ , with  $y = 36$ ,  $b = 6$  and  $x = 2$ . Match this to the logarithmic form,  $\log_b y = x$ , to find  $\log_6 36 = 2$ . Matching terms for the exponential form  $e^k = 4$  gives  $y = 4$ ,  $b = e$  and  $x = k$ . Put these values into the logarithmic form gives  $\log_e 4 = \ln 4 = k$ . ♦

↶ **Example 2.8** Rewrite  $\log_b 9 = 2$  and  $\ln k = 2$  as exponential equations.

✓ **Solution**  $\log_b 9 = 2$  is in the logarithmic form,  $\log_b y = x$ , with  $b = b$ ,  $y = 9$  and  $x = 2$ . Match this to the exponential form and find  $b^2 = 9$ . Rewrite  $\ln k = 2$  as  $\log_e k = 2$  and find that  $b = e$ ,  $y = k$  and  $x = 2$ . Put this into the exponential form and find  $e^2 = k$ . ♦

Figure 2.6 below shows the graph of  $y = \log_2 x$  and its corresponding table of values. Notice that the table of values is constructed by using the exponential form of  $y = \log_2 x$ .

□ **Figure 2.6** Graph of  $y = \log_2 x$ .



□ **Table 2.5** Logarithmic Functions

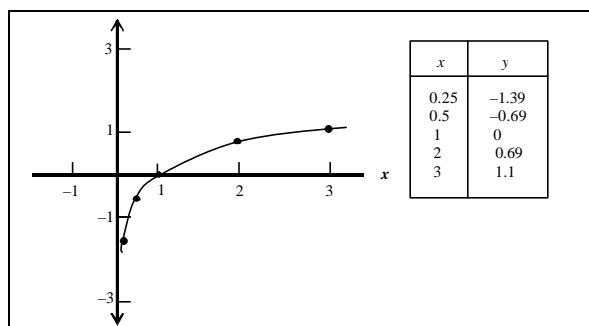
Let  $f(x) = \log_b x$ , where  $b > 0$  and  $b \neq 1$ .  
Then,

1. The domain of  $f(x)$  is  $(0, \infty)$ .
2. The range of  $f(x)$  is  $(-\infty, \infty)$ .
3. The  $x$ -intercept is  $(1, 0)$ .

↶ **Example 2.9** Graph  $f(x) = \ln x$  and verify its domain and range.

✓ **Solution** The graph is shown in Figure 2.7. The domain is  $(0, \infty)$  and the range is  $(-\infty, \infty)$ .

**Figure 2.7** Graph of  $y = \ln x$ .



To solve equations involving logarithms, we will need to know the Laws of Logarithms listed in Table 2.6.

**Table 2.6** Properties of Logarithms

If  $M$ ,  $N$ , and  $b$  are all positive numbers where  $b \neq 1$  and  $c$  is any real number, then:

- |   |                       |
|---|-----------------------|
| 1. $\log_b MN = \log_b M + \log_b N$          | 5. $\log_b 1 = 0$     |
| 2. $\log_b \frac{M}{N} = \log_b M - \log_b N$ | 6. $\log_b b^c = c$   |
| 3. $\log_b M^c = c \log_b M$                  | 7. $b^{\log_b M} = M$ |
| 4. $\log_b b = 1$                             |                       |

**Example 2.10** Given  $\log_b 4 = 0.7124$  and  $\log_b 3 = 0.5645$ , use the definition and laws of logarithms to evaluate the following expressions:

- a.  $\log_b 12$                       b.  $\log_b 9$                       c.  $\log_b \frac{16}{3}$

**Solution**

- a.  $\log_b 12 = \log_b (4 \cdot 3) = \log_b 4 + \log_b 3 = 0.7124 + 0.5645 = 1.2769$
- b.  $\log_b 9 = \log_b 3^2 = 2\log_b 3 = 2(0.5645) = 1.129$
- c.  $\log_b \left(\frac{16}{3}\right) = \log_b \left(\frac{4^2}{3}\right) = \log_b 4^2 - \log_b 3 = 2\log_b 4 - \log_b 3 = 2(0.7124) - 0.5645 = 0.8603$  ◇

**Example 2.11** Use the definition and laws of logarithms to solve the following equations for  $x$ .

- a.  $\ln(x+2) = 4$                       b.  $\log(\log 2x) = 1$

✓ **Solution**

a. First rewrite the logarithmic equation as an exponential equation then solve for  $x$ :

$$e^4 = x + 2$$

$$e^4 - 2 = x.$$

Substitute this into the original equation to check:

$$\ln(x + 2) = \ln(e^4 - 2 + 2) = \ln(e^4) = 4 \ln e = 4.$$

b. Since this is an embedded logarithm we must rewrite as an exponential equation twice, and then solve for  $x$ :

$$10^1 = \log 2x$$

$$10^{10} = 2x$$

$$\frac{10^{10}}{2} = x.$$

Check this solution in the original equation,

$$\log\left(\log\left[2 \cdot \frac{10^{10}}{2}\right]\right) = \log(\log[10^{10}]) = \log(10 \log[10]) = \log(10 \cdot 1) = 1$$

◇

↖ **Example 2.12** Solve the following exponential equations for  $x$ .

a.  $2(e^{2x-5}) = 4$

b.  $e^{\sqrt{x}} = 9$

c.  $10^{\log(2x+1)} = \ln 2$

✓ **Solution**

a. Divide both sides of the equation by 2,

$$e^{2x-5} = 2$$

Since the variable is in the exponent, rewrite as a logarithmic equation and then solve for  $x$ :

$$2x - 5 = \ln 2$$

$$2x = \ln 2 + 5$$

$$x = \frac{\ln 2 + 5}{2} \approx 2.8466.$$

b. Rewrite as a logarithmic equation,

$$\ln 9 = \sqrt{x}$$

Solve for  $x$  by squaring both sides,

$$x = (\ln 9)^2 \approx 4.8278.$$

c. Use logarithmic properties from Table 2.6 on the left side of the equation and then solve for  $x$ ,

$$2x + 1 = \ln 2$$

$$2x = \ln 2 - 1$$

$$x = \frac{\ln 2 - 1}{2} \approx -0.1534$$



## 2.4 Modeling

As we have seen in earlier sections, we can model real world data with exponential and logarithmic functions.



**Example 2.13** The data below represents the amount of goods (in millions of dollars) the United States imported from Hungary from 1993 to 2000. Find the best fitting exponential model for this data set and use the model to predict the amount of imported goods in the year 2010.

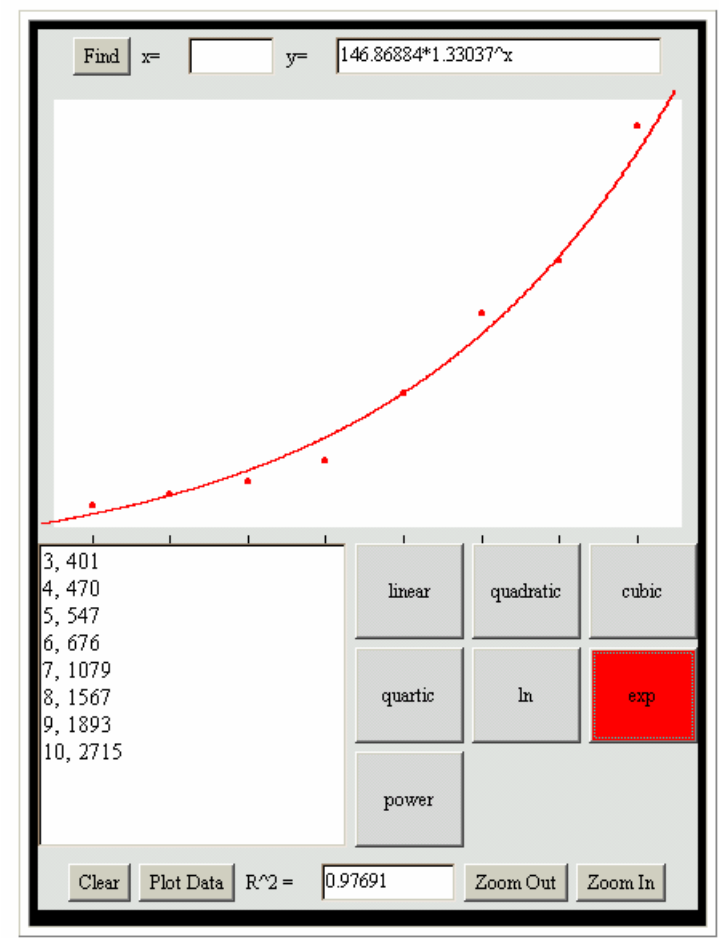
Year	1993	1994	1995	1996	1997	1998	1999	2000
Amount of goods imported (in millions of dollars)	401	470	547	676	1079	1567	1893	2715

Source: <http://www.ita.doc.gov/td/industry/otea/usfth/aggregate/HL00T11.html>



**Solution** We will make the independent variable,  $x$ , the import year where the value of  $x$  represents the number of years after 1990 (i.e. 4 represents 1994). The amount of goods imported will be the dependent variable,  $y$ . Using the regression calculator available for these modules we input the data, click on plot data, and then choose the exponential regression button. In doing so we obtain  $y = 146.87(1.33)^x$  with  $R^2 = 0.97691$  as shown in Figure 2.8.

**Figure 2.8** Modeling Applet for Example 2.13



To predict the amount of imported goods in the year 2010 we input  $x = 20$  into the regression calculator and find  $y = 21,474.84$  million dollars. ◇

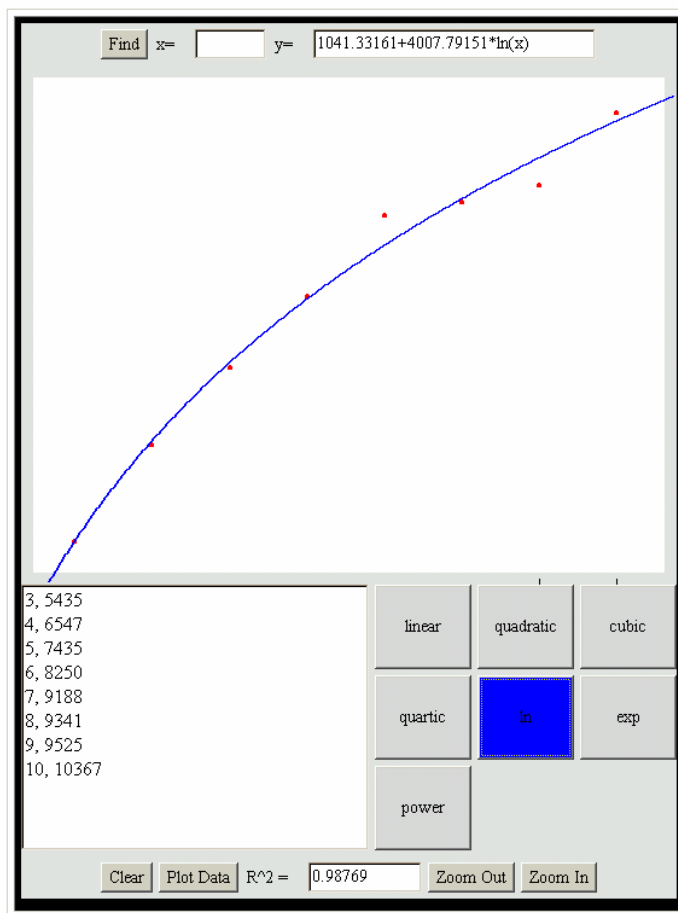
**Example 2.14** The table below represents the amount of goods (in millions of dollars) the United States imported from Indonesia from 1993 to 2000. Find the best fitting logarithmic model for this data set and use the model to predict the amount of imported goods in the year 2005.

Year	1993	1994	1995	1996	1997	1998	1999	2000
Amount of goods imported (in millions of dollars)	5435	6547	7435	8250	9188	9341	9525	10367

Source: <http://www.ita.doc.gov/td/industry/otea/usfth/aggregate/HL00T11.html>

**Solution** Using the same set up as in Example 2.13 we find the logarithmic regression model to be  $y = 1041.33 + 4007.79(\ln x)$  with  $R^2 = 0.98769$ , as shown below.

**Figure 2.9** Modeling Applet for Example 2.14



To predict the amount of imported goods in the year 2005 we input  $x = 15$  into the regression calculator and find  $y = 11,894.63$  million dollars. ◇

**Example 2.15** The table below represents the average price (in dollars) ranchers in the United States received per head of cattle from 1996 to 2001. Find the best fitting model for this data set and use the model to predict the average price per head in 1993.

Year	1996	1997	1998	1999	2000	2001
Average price paid per head of cattle (in dollars)	503	525	603	594	683	725

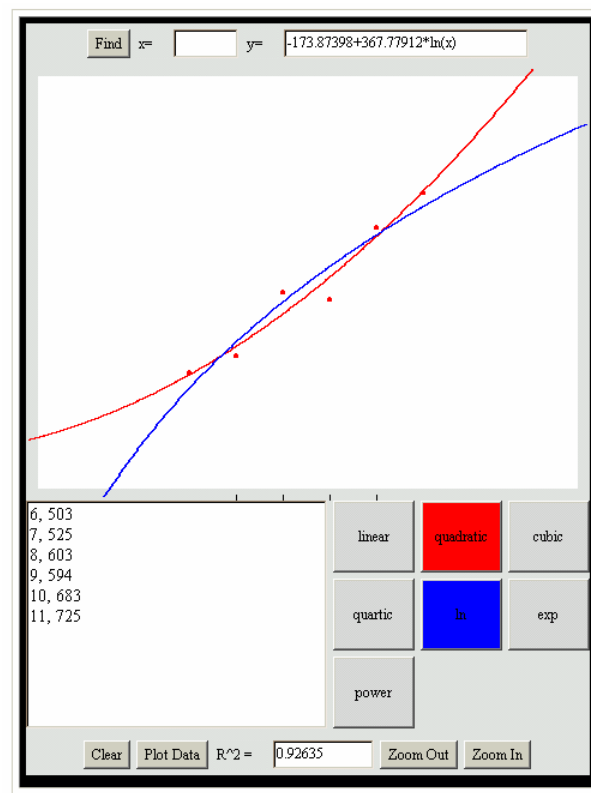
Source: <http://www.usda.gov/nass/pubs/agr01/acro01.htm> , hog/cattle/sheep

**Solution** Let  $x$  be the number of years after 1990 and  $y$  be the dollar amount paid per head of cattle. Using the regression calculator we get the following equations and  $R^2$  values.

Regression Model	Equation	R <sup>2</sup>
Exponential	$y = 319.14(1.08)^x$	0.95136
Logarithmic	$y = -173.87 + 367.78(\ln x)$	0.92635
Linear	$y = 45x + 223$	0.94724
Quadratic	$y = 2.57x^2 + 1.29x + 401.29$	0.95383
Cubic	$y = 0.37x^3 - 6.87x^2 + 79.69x + 189.73$	0.95407
Power	$y = 164.68(x^{0.61})$	0.93917

By comparing the R<sup>2</sup> values we see that the exponential, quadratic, and cubic models have the highest value. Since the cubic value is not significantly higher than the quadratic we can eliminate the cubic model because the difference in the R<sup>2</sup> value does not justify the more complicated model. Thus, we must decide between the exponential and quadratic model by studying their overall shape as seen in Figure 2.10.

 **Figure 2.10** Modeling Applet for Example 2.15



Both graphs extend to positive infinity with the same general shape, but from 0 to 5 the graphs have slightly different characteristics. It appears that the exponential model has the same general shape as the data from [0, 5] whereas the quadratic model tends to deviate from the data's shape. It is for this reason that the exponential model would best fit this data set. Now using this model to predict the price per head of cattle in 1993 we need to find  $y$  when  $x = 3$ . Using the regression calculator we find  $y \approx \$399$ .  $\diamond$



## Sample Quiz

**Question 2.1** Determine whether the following table represents an exponential function by calculating successive  $f(x)$  ratios. If it represents an exponential function, give the value of the base and state whether it is a growth or decay function.

$x$	3	4	5	6	7	8	9
$f(x)$	42	51	62	75	90	109	133

**Question 2.2** Solve  $4^x - 15 \cdot 2^x = -36$  for  $x$ .

**Question 2.3** How much money should Dave deposit into an account that earns 5.5% annual interest compounded monthly if he wants to have \$30,000 in the account 18 years from now?

**Question 2.4** After some research, you found two investment options for your \$18,000 graduation gift. Bank One offers 8% compounded semiannually and Bank One-Plus offers 7.5% compounded monthly.

- Which bank would be the better investment if you left the money in the account for 1 year?
- How many years would it take for Bank One to become the better investment?

**Question 2.5** Find the amount in an account after 7 years if \$3,000 is compounded continuously at 5.25%.

**Question 2.6** Solve  $\ln(\ln 2x) = 0$  for  $x$ .

**Question 2.7** The number of DVD players supplied to an electronics store is given by the equation  $S(p) = 150e^{0.004p}$  where  $p$  is the price in dollars. If the store is supplied 215 DVD players, at what price should they sell them?

**Question 2.8** Given  $\log_b 5 = 2.3219$ ,  $\log_b 3 = 1.5850$ , and  $\log_b 7 = 2.8074$ , find  $\log_b \left( \frac{21}{25} \right)$ .

**Question 2.9** The following table represents the assets (in billions of dollars) of FDIC insured Commercial Banks ( $y$ ) in the United States from 1987 to 2001 ( $x$ ). Find the best fitting exponential and logarithmic model, along with their  $R^2$  values, and then determine which of the two would be the better model.

$x$	1987	1988	1989	1990	1991	1992	1993	1994
$y$	2,913	3,056	3,207	3,361	3,377	3,438	3,569	3,893
$x$	1995	1996	1997	1998	1999	2000	2001	
$y$	4,171	4,397	4,771	5,181	5,469	5,983	6,360	

Source: [http://www3.fdic.gov/sod/pdf/dsta\\_2001.pdf](http://www3.fdic.gov/sod/pdf/dsta_2001.pdf)

**Question 2.10** The following table represents the assets (in billions of dollars) of FDIC insured Savings Institutions ( $y$ ) in the United States from 1987 to 2001 ( $x$ ). Find the best fitting model for this data and state why it is the best model.

x	1987	1988	1989	1990	1991	1992	1993	1994
y	1,441	1,556	1,512	1,317	1,161	1,078	1,004	999
x	1995	1996	1997	1998	1999	2000	2001	
y	1,017	1,023	1,029	1,045	1,126	1,179	1,275	

Source: [http://www3.fdic.gov/sod/pdf/dsta\\_2001.pdf](http://www3.fdic.gov/sod/pdf/dsta_2001.pdf)