Modeling Wound Healing Using Deep Learning

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Introduction

- In this research project, we use the Alternating Direction Implicit (ADI) method to solve a partial differential equation that models wound healing and also determine the necessary parameters to achieve the stability of the ADI method.
- We use the pictures of the wounds as the initial data. In addition, the task requires extracting the location and boundary of the wounds from digital photos.

Mathematical Models

We use the Heat Equation problem or specific problem (2D Heat Equation) to model the wound healing process to deal with this problem.
Consider:

$$\frac{\partial u}{\partial t} = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + f(x,y,t); \tag{1}$$

where
$(x,y) \in [0,L] * [0,L]; t > 0;$
$u = g(x,y,t); u(x,y,0) = I(x,y)$
Note: Usually, we cannot compute the solution to the equation explicitly, but we can do a numerical approximation.

Deep Learning Tools

- The problem is the segmentation problem, where we want to calculate the wound area accurately from the picture. Deep Learning is a possible way to do that when we can solve the computer vision problem by analyzing all the pictures that we have.
- Due to limitations of the ground-truth labeled data, we augment the set of pictures to a more massive test dataset.
- We have tried different models to consider the most suitable one for this segmentation problem. And we found that the two promising models are U-Net [1] and Residual U-Net [2].

Approximation Methods

To deal with the 2D-Heat Equation initial value problem, we use approximation methods and consider their stability when the spatial and time spacings approach zero.
We compare the **Forward Euler Method** and the **ADI Method**: We compare **Forward Euler Method** and **Alternating Direction Implicit Method**:

- **Forward Euler Method:** Consider the grid on $\omega = [0, L_x] * [0, L_y]$ with grid point (x_i, y_j) is introduced where $x_i = i\Delta x$ and $y_j = j\Delta y$, for $i = 0, ..., n_x, j = 0, ..., n_y$
  From (1), we have:

$$u_{i,j}^{n+1} = \frac{\Delta t}{\Delta x^2}(u_{i-1,j}^n - 2u_{i,j}^n + u_{i+1,j}^n) + \frac{\Delta t}{\Delta y^2}(u_{i,j-1}^n - 2u_{i,j}^n + u_{i,j+1}^n) + u_{i,j}^n \tag{2}$$

- **ADI Method:** Fundamental Idea is replacing a two-dimensional problem with a series of one-dimensional problems to generate a computationally efficient algorithm. We use the same grid as the Euler Method. However, we divide each time step into two steps of size $\Delta t/2$, and in each substep, we computed the solution in one dimension.

$$u_{i,j}^{n+1} = \frac{\Delta t}{2\Delta x^2}(u_{i-1,j}^{n+\frac{1}{2}} - 2u_{i,j}^{n+\frac{1}{2}} + u_{i+1,j}^{n+\frac{1}{2}} + u_{i-1,j}^{n+\frac{1}{2}} - 2u_{i,j}^{n+\frac{1}{2}} + u_{i+1,j}^{n+\frac{1}{2}}) + \frac{\Delta t}{2\Delta y^2}(u_{i,j-1}^n - 2u_{i,j}^n + u_{i,j+1}^n + u_{i,j-1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j+1}^{n+1}) + u_{i,j}^n$$

By considering the stability of two methods, we found out that the **Forward Euler Method** becomes inefficient at the ratio of $\frac{\Delta t}{\Delta x^2} \geq 0.5$. Moreover, we analyzed the **ADI method** by the von Neumann method, showing that the ADI method is unconditionally stable. Also, by working on some experiments and exact data, we see that the ADI method performs better than the Forward Euler Method for this problem.

Deep Learning Methods

We compare the accuracy and the loss between U-Net and Residual U-Net (see [1] and [2]). Although performing well on the training set, the Residual U-Net model performs poorly on the test set for this segmentation problem. Furthermore, we can easily see that the Residual U-Net model is over-fitting. The reason is the limitations of the labeled data that we have. Meanwhile, the Residual U-Net is more complex than the regular U-Net model when it includes the U-Net model and the residual neural network.

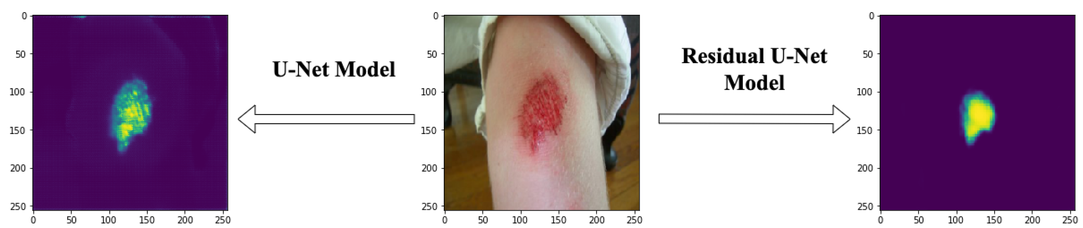

Figure 1: Comparing between U-Net Model and Residual U-Net Model on a picture from Valley Occupational Medical Center.

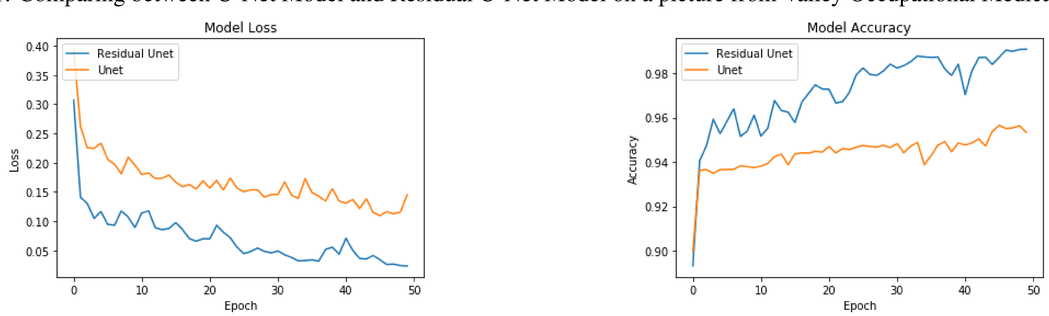

Figure 2: The loss and accuracy of U-Net and Residual U-Net models on the training set.

Results & Conclusions

- We proved the stability of the Euler Method and ADI Method for Approximation problems and we found that **ADI method** is more efficient than the Euler Method in the stability.
- We have built the program which uses the **ADI method** to model the solution for the diffusion equations.
- We analyzed the Deep Learning models which are used to capture the wound in the pictures.

However, due to the limitation of the labeled data of the wound images, our models are still extracting features ineffectively.

Future Plans

- To improve the Deep Learning models, we are collecting and making more labeled data to feed into the models as well as improving the performance of Residual U-Net, which is a promising model for the Wound Segmentation Problem.
- We seek to combine the ADI program and the Deep Learning program into one application.
- We will analyze other discrete models apart from the **ADI method** for the diffusion equations.

References

[1] Olaf Ronneberger, Philipp Fischer, and Thomas Brox. "U-net: Convolutional networks for biomedical image segmentation". In: *International Conference on Medical image computing and computer-assisted intervention*. Springer. 2015, pp. 234–241.

[2] Zhengxin Zhang, Qingjie Liu, and Yunhong Wang. "Road extraction by deep residual u-net". In: *IEEE Geoscience and Remote Sensing Letters* 15.5 (2018), pp. 749–753.