Overview

- Photoacoustic spectroscopy sensors
  - Use a quartz tuning fork and a modulated laser to detect trace gases
  - Are compact and highly sensitive
- We use a 2-way coupled model:
  - Can be used to optimize geometry
  - System of thermo-visco-acoustic equations in the fluid
  - Equations of linear elasticity in the structure
  - Interface conditions between the fluid and the structure
- Goal:
  - Estimate effect that viscous damping in the fluid has on the sensor performance
  - Demonstrate the need for two-way model by comparison to a commonly used model with one-way coupling
- We use cylindrical symmetry to obtain an analytic solution
- Reduces Helmholtz system of PDE's to ODE system

Applications

- Non-invasive disease diagnosis (e.g., lung cancer)
- Atmospheric carbon dioxide levels
- Volcanic emissions
- Detection of harmful gases
- Automobile, aircraft and marine emissions
- Homeland security (e.g., chemical weapons)

The physics of trace gas sensing

The detection of trace gases is based on the interaction between

- Optical radiation: a laser source
- Gas molecules: absorb light only at certain wavelengths

Uses both piezo- and pyroelectric properties of quartz

Piezoelectricity is the ability of a material to generate electricity in response to applied stress.

Pyroelectricity is the ability of material to generate electricity when heated.

Quartz-Enhanced PhotoAcoustic Spectroscopy acoustic wave ⇒ mechanical resonance in a QTF

Resonant OptoThermoAcoustic Detection diffusion wave ⇒ mechanical resonance in a QTF

Simplified system

- Fluid domain: An infinite cylinder of radius $R_1$
- Structural domain: An infinite annulus surrounding the fluid with inner radius, $R_1$, and the outer radius, $R_2$.

The source function:

$$ S(r, t) = \cos(\omega t) \exp(-r^2/2\sigma^2) $$

Cross section of the computational domain

2-way coupled model

- Fluid variables
  - $p$: pressure, $\tau_F$: temperature, $\mathbf{v}_F$: velocity
- Fluid equations
  - $\Delta \tau_F = -(k^2 - i\lambda)\left[\tau_F - \tau_S\right]
  - $\Omega \tau_F = -ik^2(\tau_F - p) - S$
  - $\Delta \mathbf{v}_F - i\nu \mathbf{v}_F = \nabla p$

Here $k = O(10^2)$ is the wave number, $\lambda, \Omega = O(10^{-5})$ are viscous and thermal parameters.

- Structural variables
  - $u_S$: displacement, $\tau_S$: temperature
- Structural equations
  - $-\rho_S \ddot{u}_S + \nabla \cdot C \nabla u_S = \nabla \cdot C [\alpha_S \tau_S]$
  - $i\nu \tau_S - \nabla S = 0$

Here $C$ is the elasticity tensor.

- Interface-Boundary conditions
  - $\tau_S(R_1) = \tau_F(R_1)$
  - $K_S \nabla \tau_S(R_1) \cdot \mathbf{n} = -K_F \nabla \tau_F(R_1) \cdot \mathbf{n}$
  - $\tau_S(R_2) = 0$, $u_S(R_2) = 0$.
  - $\tau_F(0) < \infty$, $p(0) < \infty$, $\nu(0) < \infty$.

- Condition on the fluid due to the structure
  - $(1 - i\nu) p(R_1) + i\nu \lambda \tau_f(R_1) = \rho_F \omega^2 u_S(R_1)$

- Condition on the structure due to the fluid
  - $(C \nabla u_S(R_1)) - C[\alpha_S \tau_S(R_1)] \cdot \mathbf{n} = -p(R_1) \mathbf{n} + \sigma_{\mathbf{j} \mathbf{k}} n$

- The viscous stress tensor
  - $\sigma_{\mathbf{j} \mathbf{k}}' = \mu_F \left( \frac{\partial u_F}{\partial t} + \mathbf{v}_F \cdot \nabla \mathbf{v}_F \right) - \frac{2}{3} \delta_{\mathbf{j} \mathbf{k}} \nabla \cdot \mathbf{v}_F + \eta_F \delta_{\mathbf{j} \mathbf{k}} \nabla \cdot \mathbf{v}_F$

- Models the damping of the tuning fork due to its motion through the fluid

1-way coupled model

- Acoustic pressure and temperature drive mechanical vibration
- No coupling from structure to fluid
- 1-way boundary conditions

$$ p'(R_2) = 0 $$

- Ad-hoc damping model

$$ \nabla \cdot C [\alpha_S \tau_S] = \nabla \cdot C [\alpha_S \tau_S] $$

Damping is incorporated into the model by adding an additional term, $-\omega \delta S \mathbf{u}_S$ in the structural displacement equation

- The parameter $\delta S$ can be calculated from the resonance curve of two-way model or, by experimentally in terms of the $Q-$ factor

One-way model can not be used to model arbitrary because, It requires measured $Q-$factor

Resonance curves at low and high ambient pressure

- Resonance curves agree much better at high ambient pressure than low ambient pressure

1-way versus 2-way signal strength

- One possible reason for lack of agreement is
  - In one-way coupled damping is distributed over the entire structure
  - In two-way coupled damping is incorporated only at the inner boundary of the structure

References


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