A two way coupled model for viscous damping of a vibrating structure with visco-thermo-acoustic forcing

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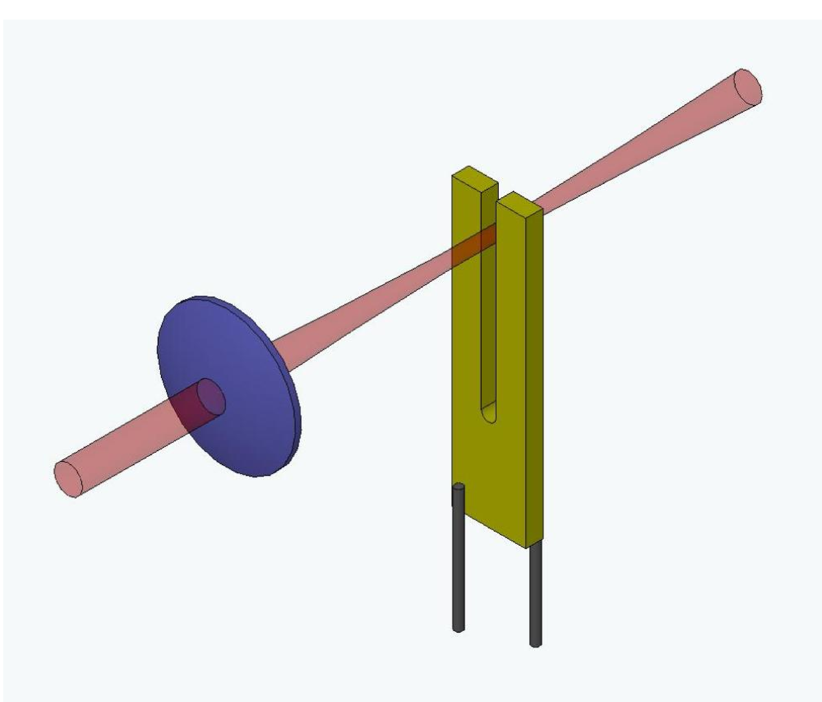
Overview

- **Photoacoustic spectroscopy sensors**
  - Use a quartz tuning fork and a modulated laser to detect trace gases
  - Are compact and highly sensitive
- **We use a 2−way coupled model:**
  - Can be used to optimize geometry
  - System of thermo-visco-acoustic equations in the fluid
  - Equations of linear elasticity in the structure
  - Interface conditions between the fluid and the structure
- **Goal:**
  - Estimate effect that viscous damping in the fluid has on the sensor performance
  - Demonstrate the need for two-way model by comparison to a commonly used model with one-way coupling
- We use cylindrical symmetry to obtain an analytic solution
  - Reduces Helmholtz system of PDE's to ODE system

Applications

- Non-invasive disease diagnosis (e.g. for lung cancer)
- Atmospheric carbon dioxide levels
- Volcanic emissions
- Detection of harmful gases
- Automobile, aircraft and marine emissions
- Homeland security (e.g., chemical weapons)



Experimental configuration of a trace gas sensor

The physics of trace gas sensing

The detection of trace gases is based on the interaction between

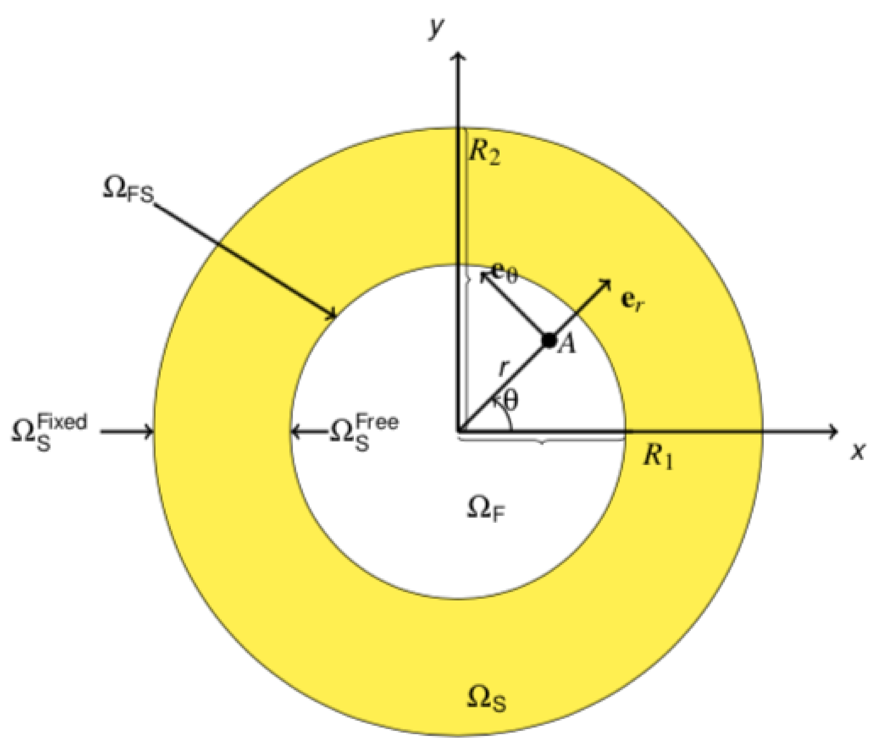
- **Optical radiation**: a laser source
- **Gas molecules**: absorb light only at certain wavelengths

- Employs both piezo- and pyroelectric properties of quartz
- Piezoelectricity is the ability of a material to generate electricity in response to applied stress.
- Pyroelectricity is the ability of material to generate electricity when heated.
- **Q**uartz-**E**nhanced **P**hoto**A**coustic **S**pectroscopy acoustic wave ⇒ mechanical resonance in a QTF
- **R**esonant **O**pto**T**hermo**A**coustic **DE**tection diffusion wave ⇒ mechanical resonance in a QTF

Simplified system

- **Fluid domain:** An infinite cylinder of radius R_1
- **Structural domain:** An infinite annulus surrounding the fluid with inner radius, R_1, and the outer radius, R_2.
- **The source function:**

$$S(r,t) = \cos(\omega t)\exp\left(-r^2/2\sigma^2\right)$$



Cross section of the computational domain

2-way coupled model

- **Fluid variables**
  - p = pressure, τ_F = temperature, $\mathbf{v}_F$ = velocity
- **Fluid equations**
  - $\Delta p = -(k^2 - i\Lambda\Delta)(p - \tau_F)$
  - $\Omega\Delta\tau_F = -ik^2(\tau_F - p) - S$
  - $\Delta\mathbf{v}_F - i\,\mathbf{v}_F = \nabla p$

  Here $k = O(10^2)$ is the wave number, Λ, $\Omega = O(10^{-5})$ are viscous and thermal parameters
- **Structural variables**
  - $\mathbf{u}_S$ = displacement, τ_S = temperature
- **Structural equations**
  - $-\rho_S\omega^2\mathbf{u}_S + \nabla\cdot C[\nabla\mathbf{u}_S] = \nabla\cdot C[\alpha_S\tau_S]$
  - $i\,\omega\tau_S - D_S\Delta\tau_S = 0$

  Here C is the elasticity tensor
- **Interface-Boundary conditions**

$$\tau_S(R_1) = \tau_F(R_1),$$
$$K_S\nabla\tau_S(R_1)\cdot\mathbf{n} = -K_F\nabla\tau_F(R_1)\cdot\mathbf{n},$$
$$\tau_S(R_2) = 0,\quad u_S(R_2) = 0,$$
$$\tau_F(0) < \infty,\ p(0) < \infty,\ v_F(0) < \infty.$$

  - Condition on the fluid due to the structure

$$(1 - i\gamma\Lambda)\,p'(R_1) + i\alpha\gamma\Lambda\,\tau_F'(R_1) = \rho_F\,\omega^2\,u_S(R_1),$$

  - Condition on the structure due to the fluid

$$(C[\nabla\mathbf{u}_S(R_1)] - C[\alpha_S\tau_S(R_1)])\,\mathbf{n} = -p(R_1)\mathbf{n} + \sigma_F'\mathbf{n}$$

- **The viscous stress tensor**

$$\sigma'_{jk} = \mu_F\left(\frac{\partial v_j}{\partial x_k} + \frac{\partial v_k}{\partial x_j} - \frac{2}{3}\delta_{jk}\nabla\cdot\mathbf{v}_F\right) + \eta_F\delta_{jk}\nabla\cdot\mathbf{v}_F$$

  - Models the damping of the tuning fork due to its motion through the fluid

1-way coupled model

- Acoustic pressure and temperature drive mechanical vibration
- No coupling from structure to fluid
- 1-way boundary conditions

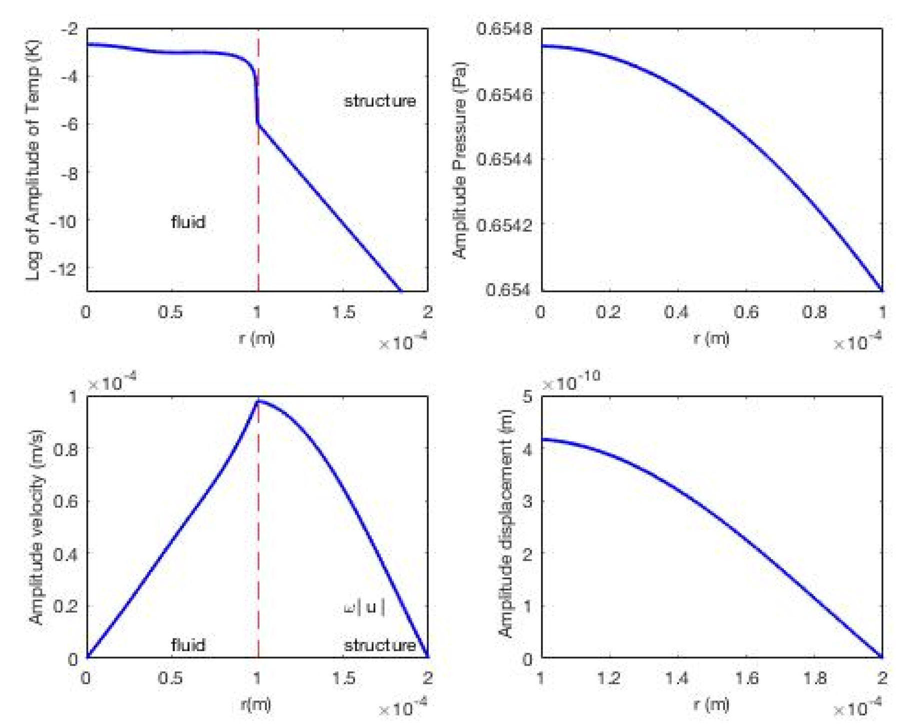
$$\left(C[\nabla\mathbf{u}(R_1)] - C[\alpha_S\tau_S(R_1)]\right)\mathbf{n} = -p(R_1)$$
$$p'(R_1) = 0$$

- Ad-hoc damping model

$$\nabla\cdot C[\nabla\mathbf{u}_S] + (\rho_S\omega^2 - i\,\omega\,\delta_S)\,\mathbf{u}_S = \nabla\cdot C[\alpha_s\tau_S]$$

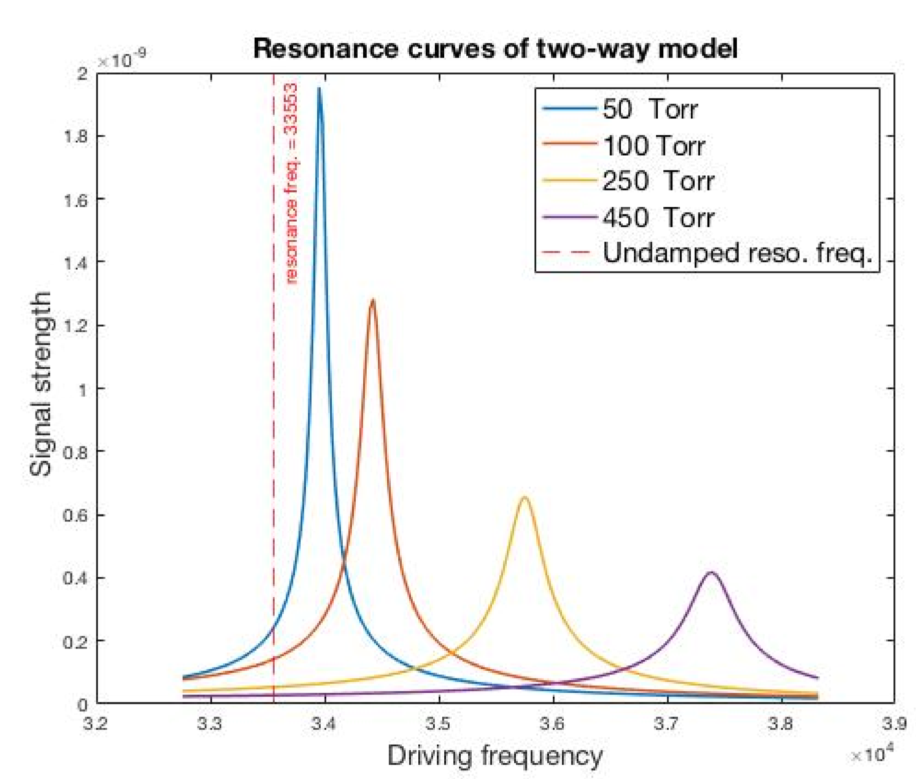
- Damping is incorporated into the model by adding an additional term, $-i\omega\,\delta_S\,\mathbf{u}_S$ in the structural displacement equation
  - The parameter δ_S can be calculated from the resonance curve of two-way model or, by experimentally in terms of the $Q-$ factor
  - The $Q-$factor quantifies how effectively the resonator resists energy losses due to damping
- One-way model can not be used to model arbitrary because, It requires measured Q-factor

Solutions of 2-way coupled model



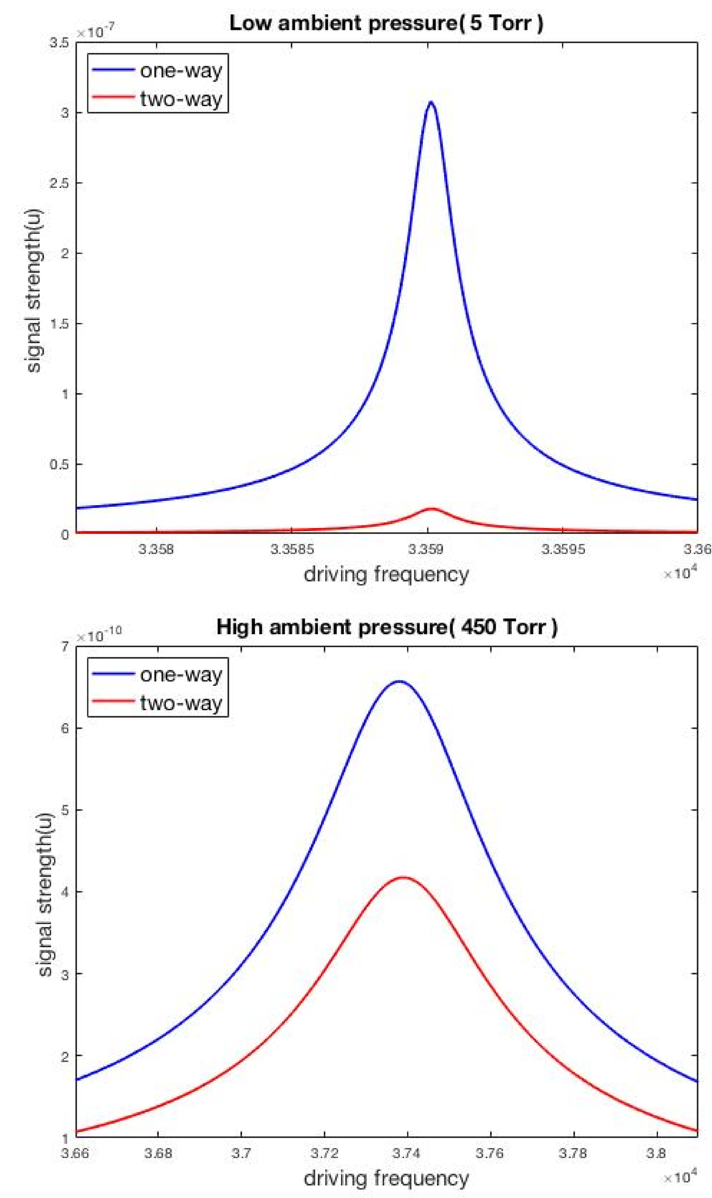
τ_F, p, v_F, and u_S of the two-way coupled model at 450 Torr

2-way resonance curves



Two-way resonance curves for varying ambient pressure

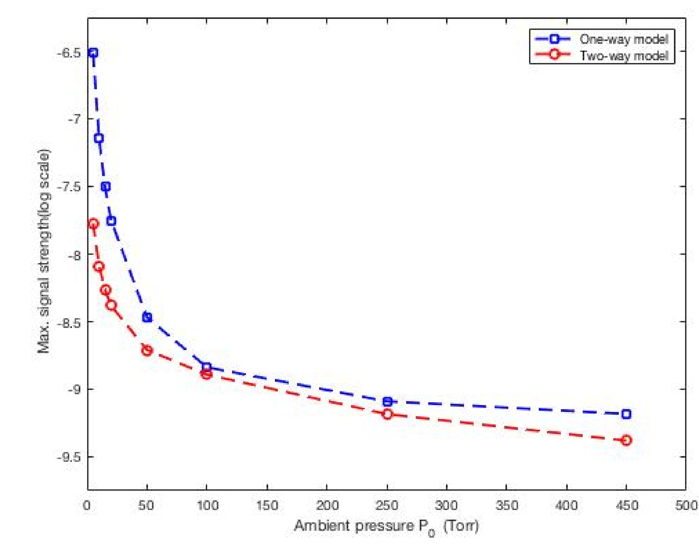
Resonance at low and high pressure



Resonance curves at low and high ambient pressure

- Resonance curves agree much better at high ambient pressure than low ambient pressure

1-way verses 2-way signal strength



Ambient pressure verses Max. signal strength

- One possible reason for lack of agreement is
  - In one-way coupling damping is distributed over the entire structure
  - In two-way coupling damping is incorporated only at the inner boundary of the structure

References

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