On Sinc Methods and Photonic Crystal Fibers

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Objectives

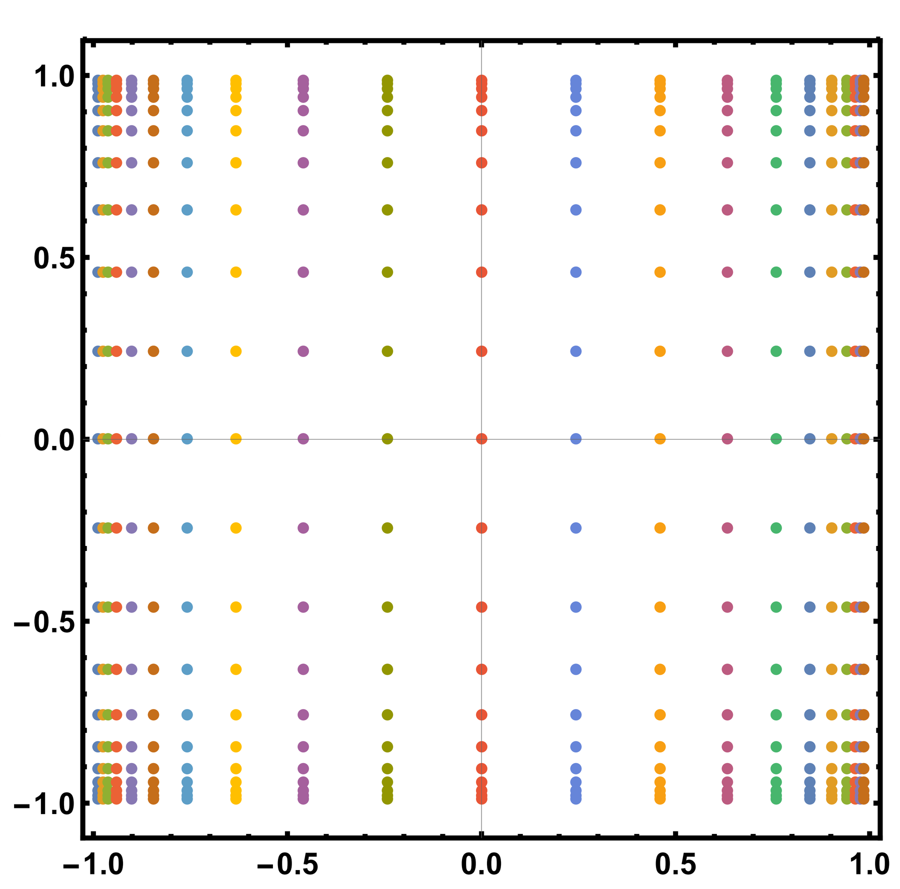
The study aims to compare and measure the approximate estimated differences in two computational approaches. In other words, interpreting the differences based on two algorithms computing the Eigen-mode. In addition, proposing a Photonic Crystal Fiber (PCF) novel model for research purposes and numerical studies.

Introduction

Modeling Photonic Crystal Fibers (PCFs) is challenging due to presence of sharp edges arising from air holes. Hollow-core PCF (**HCPCF**) is modeled in finite element method (**FEM**) using Hilbert space functions via Modeling Software. FEM results are compared with Collocation Scheme using Sinc functions (**SC**) via Wolfram Mathematica program. The study investigates effect of reducing the number of sampling data used in the overall application discretization and approximations, referred to by Sinc points with positive non-equidistant step size

$$h = \frac{\pi}{\alpha\sqrt{N}}, \tag{1}$$

For positive α, $N \in \mathbb{N}$. Collocating the Helmholtz partial differential equation with nonlinear potential function $V(x, y)$, which proposes a novel numerical model for the HC-PCF in different crystal lattice.



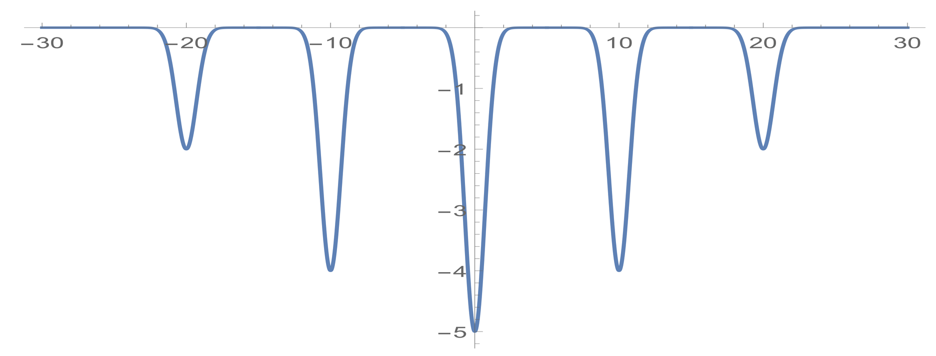
SC Sampling Sinc Points Distribution N=10, α=2

HC Photonic Crystal Fibers

Light transmission in HCPCFs is governed by Helmholtz eigenvalue problem of electromagnetism.

$$-\partial_{xx}u(x,y)-\partial_{yy}u(x,y)+V(x,y)u(x,y) = \lambda u(x,y) \tag{2}$$

HCPCF sharp steepness of air holes edges due to difference in refractive indices between air and background material, which act along with the lattice structure as **Photonic Bandgap guidance**. Sharp steepness is modeled by superimposing Gaussian functions constructing the lattice structure in the 2D. Developed model is fed to the Helmholtz's potential.



Superimposing Gaussian functions in 1D

Sinc Methods

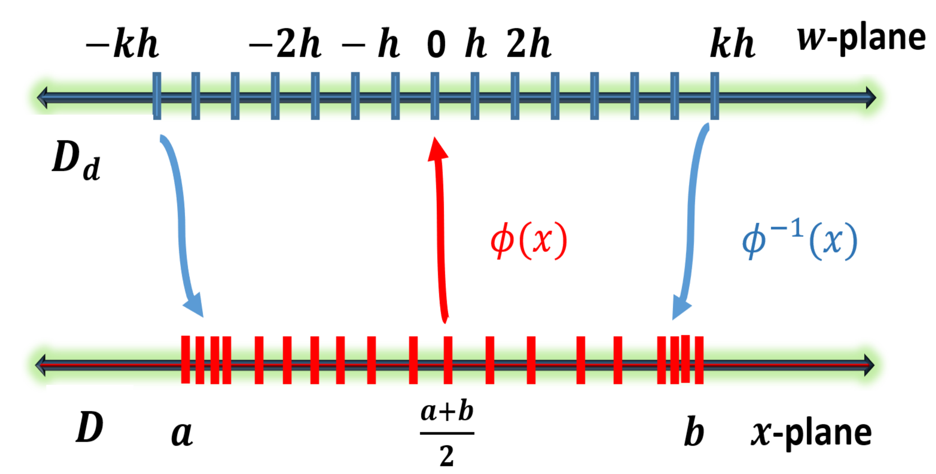
Non-equidistant Sequence of Sinc points is generated using a conformal map over a finite interval [a,b] that redistributes the infinite number of equidistant points on the real line to a finite interval. Large dense occur near end-points.

$$C_m(u,h) = \sum_{j=-N}^{N}\sum_{k=-N}^{N} u(kh) S_k(h,\phi(x_k) S_j(h,\phi(y_j) \tag{6}$$

$m = 2N+1, N \in \mathbb{N}$. The conformal map $\phi(x) = ln(\frac{x-a}{b-x})$ yields interpolation over a contour $\Gamma = \psi(\mathbb{R})$, with $a = \phi^{-1}(-\infty)$ and $b = \phi^{-1}(\infty)$ denote the end points of Γ at Sinc points $\{x_k\}_{k=-N}^{N}$ on [a,b].

$$x_k = \phi^{-1}(kh) = \psi(kh) = \frac{a+be^{kh}}{1+e^{kh}} \tag{7}$$

Same formula for y_j. Finally vectors of values of functions are computed at Sinc points for positive h. k and j are $\pm1, \pm2, \ldots$. In addition S is row vector of scaled-shifted basis functions and an operator $V_m(u)$ that maps u into a column vector of order m.



Conformal Mapping

This notation enables us to write the interpolation scheme 6 in simple operator form.

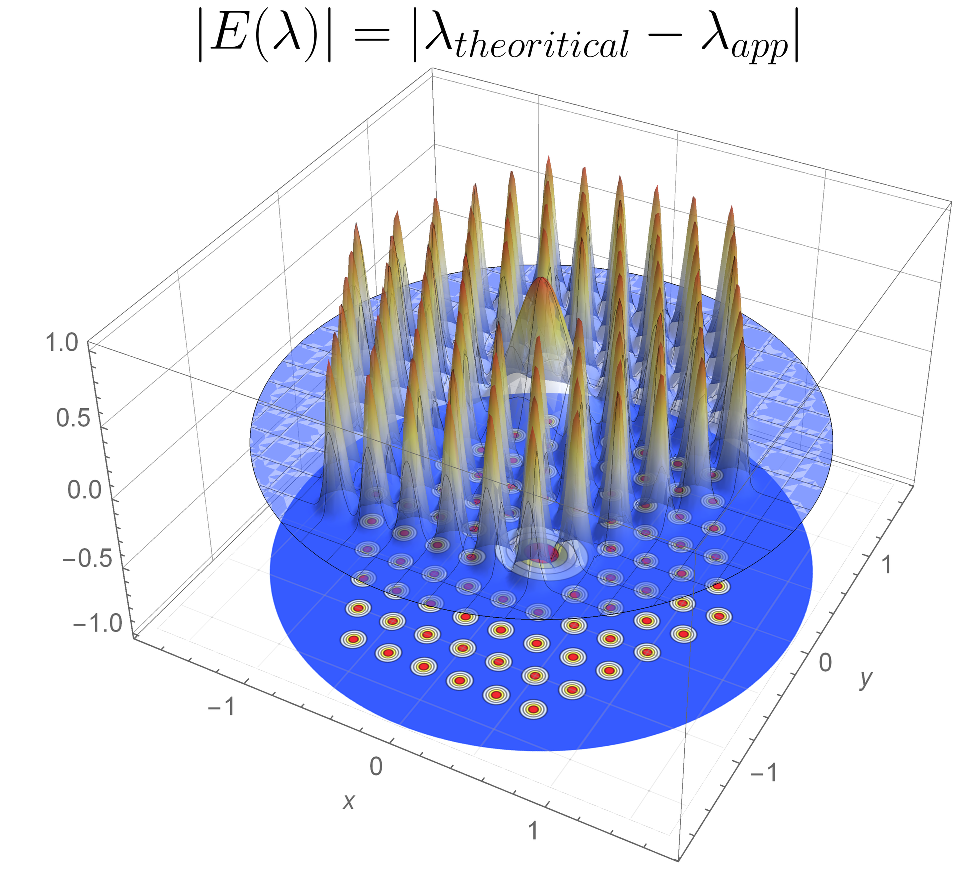
$$S = (S_{-N}, \ldots, S_N), \tag{3}$$

$$V_m(u) = (u(z_{-N}, \ldots, u(z_N)))^T \tag{4}$$

Results

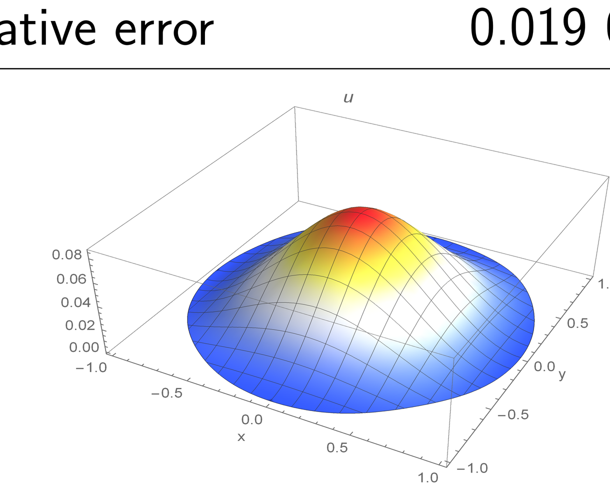
In error bounds measurements, stabilization and convergence curves are studied for the SC implementation.

$$|E(\lambda)| = |\lambda_{theoritical} - \lambda_{app}| \tag{5}$$

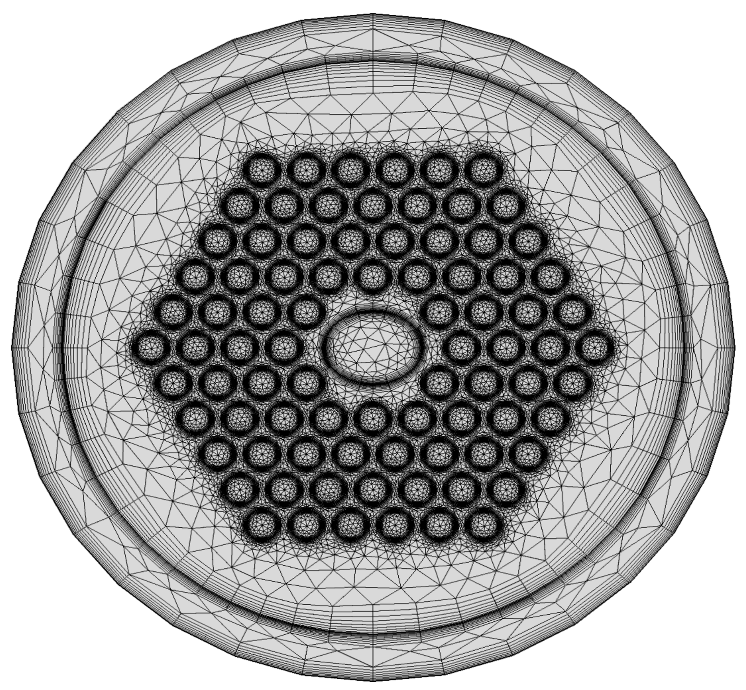


SC Novel Numerical PCF Model Structure 3D Plot

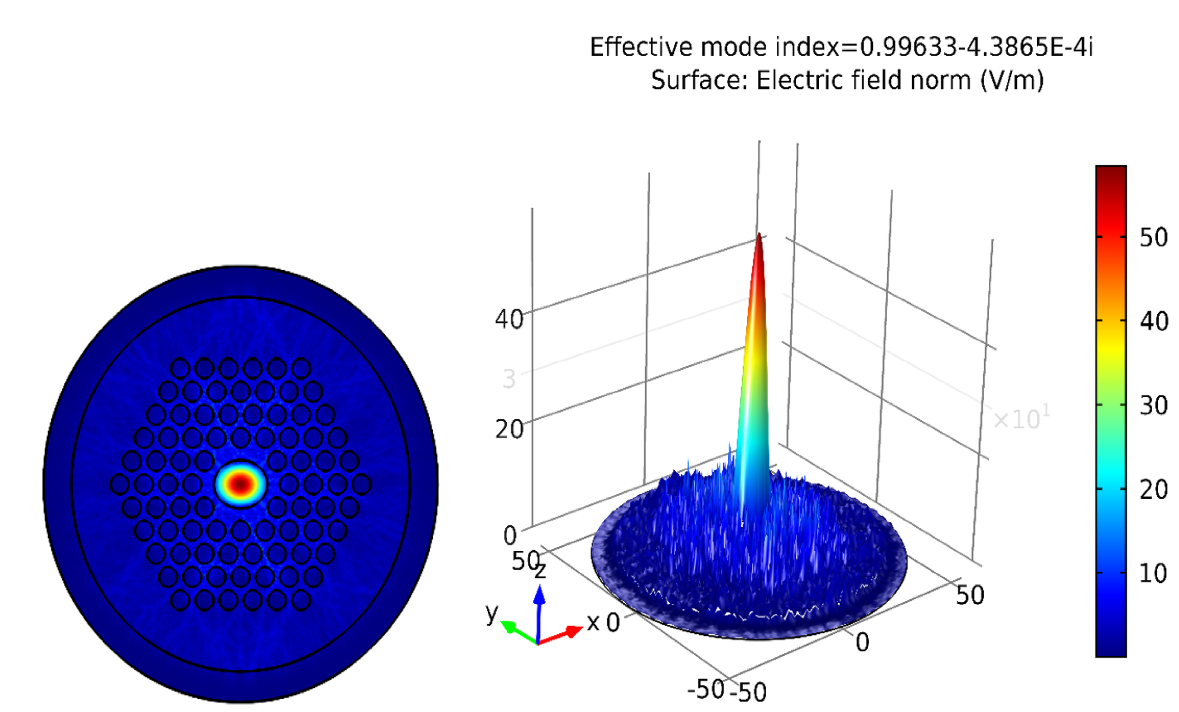
	SC	FEM
Fundamental Eigenvalue	3.97	3.96
Absolute error	0.08	0.09
Relative error	0.019	0.022



SC Fundamental Eigen-function 3D plot



FEM Mixed Mesh Elements Coarse Size



2D, 3D FEM PCF Surface Plots

Conclusion

SC approach achieves the FEM results with a comparable accuracy using few number of Sinc points. In addition, SC rapid **exponential** convergence regardless the discretization quantity, makes it an attractive alternative to traditional multi-grid computational methods. Moreover, Dynamic interactivity to the *core technique* has the advantage that error decay is estimated.

References

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