

MATH 151, SPRING 2022  
COMMON EXAM II - VERSION **A**

LAST NAME(print): \_\_\_\_\_ FIRST NAME(print): \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

SECTION NUMBER: \_\_\_\_\_

**DIRECTIONS:**

1. No calculator, cell phones, or other electronic devices may be used, and they must all be put away out of sight.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*
4. In Part 2, present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. **Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.**

THE AGGIE CODE OF HONOR

**“An Aggie does not lie, cheat or steal, or tolerate those who do.”**

Signature: \_\_\_\_\_

**PART I: Multiple Choice. 3 points each**

1. Find the equation of the tangent line to the graph of  $f(x) = \frac{2x}{3x+1}$  at  $x = 1$ .

(a)  $y - \frac{1}{2} = -\frac{1}{8}(x - 1)$

(b)  $y - \frac{1}{2} = \frac{7}{8}(x - 1)$

(c)  $y - \frac{1}{2} = -\frac{2}{3}(x - 1)$

(d)  $y - \frac{1}{2} = \frac{1}{8}(x - 1)$

(e)  $y - \frac{1}{2} = -\frac{7}{8}(x - 1)$

2. Find the equation of the tangent line to the graph of  $x = 3t^2 - 2t + 1$ ,  $y = t^2 + 1$  at the point where  $t = 1$ .

(a)  $y = \frac{1}{2}x + 1$

(b)  $y = \frac{1}{2}x - 1$

(c)  $y = 2x - 2$

(d)  $y = x$

(e) None of these

3. Find the slope of the tangent line to the curve  $x^2 - y^2 = 16$  at the point  $(-5, 3)$ .

(a)  $-\frac{5}{3}$

(b)  $\frac{5}{3}$

(c)  $-\frac{5}{4}$

(d)  $\frac{4}{5}$

(e)  $\frac{3}{4}$

4. Determine where the function  $f$  below is NOT differentiable.

$$f(x) = \begin{cases} \sin x & x < 0 \\ -x^3 + x & 0 \leq x < 1 \\ x^2 - 1 & 1 \leq x \end{cases}$$

- (a)  $x = 0$  only
- (b)  $x = 1$  only
- (c)  $x = 0$  and  $x = 1$
- (d)  $f$  is differentiable everywhere
- (e)  $f$  is not differentiable anywhere

Suppose  $f$  and  $g$  are differentiable functions which satisfy the following condition. Use the following table for Problems 5 and 6.

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	-2	1	1	1
1	-1	3	3	0

5. Let  $u(x) = \frac{f(x)}{g(x)}$ . Find  $u'(1)$ .

- (a) -1
- (b) 0
- (c) 1
- (d) 2
- (e) 3

6. Let  $v(x) = g(f(x))$ . Find  $v'(1)$ .

- (a) -2
- (b) -1
- (c) 0
- (d) 3
- (e) 5

7. Find the derivative of the function  $f(x) = \arcsin(e^{5x})$ .

(a)  $\frac{5e^{5x}}{1 + e^{10x}}$

(b)  $\frac{1}{\sqrt{1 - e^{10x}}}$

(c)  $-\frac{5e^{5x}}{\sqrt{1 - e^{10x}}}$

(d)  $-\frac{5e^{5x}}{1 + e^{10x}}$

(e)  $\frac{5e^{5x}}{\sqrt{1 - e^{10x}}}$

8. Find all point(s) on the curve parametrized by  $x = t^2 - 2t - 3$ ,  $y = t^3 - 3t^2$  where the tangent line is horizontal.

(a)  $(-4, -2)$

(b)  $(0, 0)$  and  $(-3, 0)$

(c)  $(-3, -4)$  and  $(-3, 0)$

(d)  $(0, 0)$  and  $(0, -4)$

(e) None of these

9. Find all point(s) on the curve parametrized by  $x = t^2 - 2t - 3$ ,  $y = t^3 - 3t^2$  where the tangent line is vertical.

(a)  $(-4, -2)$

(b)  $(0, 0)$  and  $(-3, 0)$

(c)  $(-3, -4)$  and  $(-3, 0)$

(d)  $(0, 0)$  and  $(0, -4)$

(e) None of these

10. Find  $f'(0)$  for  $f(x) = 2^{(3^x)}$ .

- (a)  $\ln 2 \cdot \ln 3$
- (b)  $2 \ln 2 \cdot \ln 3$
- (c)  $3 \ln 2 \cdot \ln 3$
- (d)  $8 \ln 2 \cdot \ln 3$
- (e)  $24 \ln 2 \cdot \ln 3$

11. Use the linearization of  $f(x) = \sqrt{x}$  at  $x = 9$  to estimate the value of  $\sqrt{9.1}$ .

- (a)  $\frac{179}{60}$
- (b)  $\frac{19}{6}$
- (c)  $\frac{17}{6}$
- (d)  $\frac{31}{30}$
- (e)  $\frac{181}{60}$

12. The length of a rectangle is increasing at a rate of 5 cm/sec and its width is decreasing at a rate of 4 cm/sec. When the length is 20 cm and the width is 10 cm, what is the rate of change of the area of the rectangle?

- (a)  $30 \text{ cm}^2/\text{sec}$
- (b)  $-30 \text{ cm}^2/\text{sec}$
- (c)  $60 \text{ cm}^2/\text{sec}$
- (d)  $-60 \text{ cm}^2/\text{sec}$
- (e)  $130 \text{ cm}^2/\text{sec}$

13. Find the 2021<sup>st</sup> derivative of  $f(x) = 2 \cos(2x)$

- (a)  $-2^{2021} \sin(2x)$
- (b)  $2^{2022} \sin(2x)$
- (c)  $2^{2022} \cos(2x)$
- (d)  $-2^{2021} \cos(2x)$
- (e)  $-2^{2022} \sin(2x)$

14. The position of a particle is given by the vector function  $\mathbf{r}(t) = \langle te^t, t^3 \rangle$ . Find the acceleration vector of the particle at time  $t = 1$ .

- (a)  $\langle e, 6 \rangle$
- (b)  $\langle 2e, 6 \rangle$
- (c)  $\langle 2e, 3 \rangle$
- (d)  $\langle e, 3 \rangle$
- (e)  $\langle 3e, 6 \rangle$

15. The radius of a circle was measured to be 5 ft with a maximum possible error of 0.2 ft. Use differentials to estimate the maximum possible error in the calculated area of the circle.

- (a)  $10\pi$
- (b)  $2\pi$
- (c)  $0.9\pi$
- (d)  $0.09\pi$
- (e)  $\frac{2\pi}{25}$

16. A ball is tossed in the air, and the height of the ball at time  $t$  seconds is given by  $h(t) = 25 + 10t - t^2$ , where  $h(t)$  is measured in feet from the ground. Find the maximum height  $H$  of the ball and the time  $T$  when it hits the maximum height.

(a)  $H = 25, T = 5$

(b)  $H = 50, T = 5$

(c)  $H = 50, T = 10$

(d)  $H = 25, T = 10$

(e)  $H = 30, T = 5$

17. A particle moves according to the equation of motion  $s(t) = t^2 - 2t + 3$  where  $s(t)$  is measured in meter and  $t$  is measured in seconds. Find the total distance traveled in the first 3 seconds.

(a) 2 m

(b) 3 m

(c) 4 m

(d) 5 m

(e) 6 m

18. A bacteria culture starts with 2 million bacteria and the population triples every 30 minutes. Find the number of bacteria after 90 minutes.

(a) 16 million

(b) 18 million

(c) 30 million

(d) 54 million

(e) 72 million

19. Find  $h''(1)$  if  $h(x) = e^{-x^2}$

(a)  $\frac{4}{e}$

(b)  $\frac{2}{e}$

(c)  $-\frac{2}{e}$

(d)  $\frac{1}{e}$

(e)  $-\frac{4}{e}$

20. Find  $f'(x)$  for  $f(x) = \ln\left(\frac{\sqrt{x^6+1}}{\sec^{10}x}\right)$  [HINT: Use properties of logarithms.]

(a)  $\frac{1}{12x^5} - \frac{10}{\sec x \tan x}$

(b)  $\frac{1}{12x^5} - \frac{1}{10 \sec x \tan x}$

(c)  $\frac{3x^5}{x^6+1} - \frac{10 \tan x}{\sec x}$

(d)  $\frac{3x^5}{x^6+1} - 10 \tan x$

(e)  $\frac{3x^5}{x^6+1} - \tan x$



## PART II WORK OUT

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

21. (12 points) Find  $\frac{dy}{dx}$  for the following functions. **Do not simplify after taking the derivative.**

(a)  $y = \sec^5(4x^3 + \pi^2)$

(b)  $y = \frac{2^x \tan x}{x^3}$

22. (6 points) Find  $\frac{dy}{dx}$ .

$$y = e^{5xy}$$

23. (10 points) Find  $\frac{dy}{dx}$ . **Your answer must be a function in  $x$  only.**

$$y = (\cos(4x))^{\sqrt{x}}$$

24. (12 points) Water is pumped into an inverted conical tank at a constant rate of  $3 \text{ m}^3/\text{min}$ . The tank has a height of 10 m, and the diameter across the top is 10 m. How fast is the water level rising when the water is 2 m deep? (The volume of a cone is  $V = \frac{1}{3}\pi r^2 h$ ).

**DO NOT WRITE IN THIS TABLE.**

Question	Points Awarded	Points
1-20		60
21		12
22		6
23		10
24		12
		100