Worksheet 5

1. The recursive sequence given below is bounded and increasing. Find the limit of the sequence, if it exists.

$$a_1 = 4, \quad a_{n+1} = 8 - \frac{15}{a_n}$$

   Explain clearly how you are using the hypothesis.

2. Determine if the following series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^n$$

3. Suppose the nth partial sum S_n, of the series $\sum_{k=1}^{\infty} a_k$, is equal to $4 - n3^{-n}$. Then, what is the formula for a_k and the SUM of the series ($\sum_{k=1}^{\infty} a_k = $) ?

4. Find the SUM of the geometric series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}3^{3n-5}}{7^{2n-2}}$$

5. Find the SUM of the following series

$$\sum_{n=2}^{\infty} \arctan \left(\frac{1}{n^2 + n + 1} \right)$$

   (*Hint:* $\frac{1}{n^2+n+1} = \frac{1}{n(n+1)+1} = \frac{(n+1)-n}{n(n+1)+1}$)