Worksheet 6

1. Find the limit of the sequence.

$$\left\{\sqrt{2},\sqrt{2\sqrt{2}},\sqrt{2\sqrt{2\sqrt{2}}},\cdots\right\}$$

2. Show that the sequence defined by

$$a_1 = 2; \quad a_{n+1} = \frac{1}{3 - a_n}$$

satisfies $0 < a_n \leq 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.

3. Let $\{a_n\}$ be the sequence defined by

$$a_n = \frac{\sqrt{n+1}}{5n+3}$$

Determine if the sequence is increasing/decreasing/not monotone.

- 4. Determine whether the following sequences are convergent or not. In the case, a sequence is convergent, then find its limit.
 - $a) \quad a_n = \left\{\sqrt{\frac{1+4n^2}{1+n^2}}\right\}$
 - $b) \quad a_n = \left\{ \frac{2}{\sqrt{n^3 + n}} \right\}$
 - c) $a_n = \left\{ \frac{(2n-1)!}{(2n+1)!} \right\}$
- 5. Using the integral test determine if the following series is convergent or divergent.

$$\sum_{n=2}^{\infty} \frac{1}{n l n^2(n)}$$

6. Determine whether the following series is convergent or not. If it is convergent, then find its Sum.

$$\sum_{n=1}^{\infty} \frac{1}{1 + (\frac{2}{3})^n}$$