INSTRUCTIONS

1. In Part 1 (Problems 1–10), mark your responses on your ScanTron form using a No: 2 pencil. *For your own record, mark your choices on the exam as well.*

2. Calculators **should not be used** throughout the examination.

3. In Part 2 (Problems 11–16), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly.** You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form.**
Part 1 – Multiple Choice (50 points)

Each question is worth 5 points. Mark your responses on the ScanTron form and on the exam itself.

1. Compute the indefinite integral \( \int x^3 \sqrt{2 + x^4} \, dx \).

   (a) \( \frac{2(2 + x^4)^{3/2}}{3} + C \)
   
   (b) \( \frac{(2 + x^4)^{3/2}}{4} + C \)
   
   (c) \( \frac{(2 + x^4)^{3/2}}{6} + C \)
   
   (d) \( \frac{(2 + x^4)^{-1/2}}{8} + C \)
   
   (e) \( \frac{(2 + x^4)^{-1/2}}{2} + C \)

2. Compute the indefinite integral \( \int e^x (1 + e^x)^{10} \, dx \).

   (a) \( e^x \frac{(1 + e^x)^{11}}{11} + C \)
   
   (b) \( (1 + e^x)^{11} + C \)
   
   (c) \( 10(1 + e^x)^9 + C \)
   
   (d) \( \frac{(1 + e^x)^{11}}{11} + C \)
   
   (e) \( e^x + e^{10x} + C \)
3. Evaluate the definite integral \( \int_0^{\pi/4} \frac{\sec^2 \theta}{2 + \tan \theta} d\theta \).

(a) \( \ln \left( \frac{3}{2} \right) \)

(b) \( \ln \left( \frac{4}{3} \right) \)

(c) \( \ln \left( \frac{\pi}{4} \right) \)

(d) \( \ln \left( \frac{\pi}{8} \right) \)

(e) \( \ln \left( \frac{\pi}{12} \right) \)

4. Determine the value of the positive number \( b \) for which the average value of the function \( f(x) = 2 + 6x \) on the interval \([0, b]\) is 3.

(a) \( \frac{1}{3} \)

(b) \( 3 \)

(c) \( \frac{\sqrt{10} - 1}{3} \)

(d) \( 2 \)

(e) \( \frac{1}{6} \)

5. Compute the indefinite integral \( \int xe^{-2x} \, dx \).

(a) \( \frac{x^2 e^{-2x}}{2} + C \)

(b) \( -\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9} + C \)

(c) \( xe^{-2x} + e^{-2x} + C \)

(d) \( xe^{-3x} + e^{-3x} + C \)

(e) \( -\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4} + C \)
6. Compute the indefinite integral $\int 2\sin^2 \theta \, d\theta$.

(a) $\frac{\theta}{2} + \frac{\sin(2\theta)}{4} + C$

(b) $\theta + \frac{\sin(2\theta)}{2} + C$

(c) $\frac{\theta}{2} - \frac{\sin(2\theta)}{4} + C$

(d) $\theta - \frac{\sin(2\theta)}{2} + C$

(e) $\theta + 2\sin(2\theta) + C$

7. Calculate the area of the region enclosed by the $x$-axis, $y = \ln x$, $x = e$, and $x = e^2$.

(a) $2e^3$

(b) $e^2$

(c) $\frac{1}{e^2} - \frac{1}{e}$

(d) $\frac{1}{e^3} - \frac{1}{e}$

(e) $\frac{3}{2}$

8. An aquarium 2 m long, 1 m wide, and 1 m deep is full of water. Find the work needed to pump half the water out of the aquarium. (The density of water, $\rho$, is 1000 kg/m$^3$, and acceleration due to gravity, $g$, is 9.8 m/s$^2$.)

(a) 4.9 J

(b) $9.8 \times 10^3$ J

(c) 2.45 J

(d) 250 J

(e) $2.45 \times 10^3$ J
9. Let $R$ denote the region enclosed by the $y$-axis, the line $y = 1$, and the curve $y = \sqrt{x}$. Compute the volume of the solid whose base is $R$ and whose cross sections perpendicular to the $y$-axis are semicircles.

(a) $\pi/24$

(b) $\pi/10$

(c) $\pi/20$

(d) $\pi/40$

(e) $\pi/2$

10. Suppose that $f$ is continuous on $(-\infty, \infty)$, and that $F$ is an antiderivative of $f$ in $(-\infty, \infty)$. Which of following is an antiderivative of the function $g(x) = f(2x - 3)$?

(a) $F(2x - 3)$

(b) $\frac{F(3x - 2)}{3}$

(c) $\frac{F(2x - 3)}{2}$

(d) $F(3x - 2)$

(e) insufficient information to make a determination
Part 2 (56 points)

Present your solutions to the following problems (11–16) in the space provided. Show all your work neatly and concisely, and indicate your final answer clearly. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

11. (10 points) Compute the following integral:

\[ \int \tan^3 x \sec^3 x \, dx \]
12. (10 points) Compute the following integral:

$$\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} \, dx$$
13. (10 points) Let $R$ denote the region bounded by the parabola $y = 1 - x^2$ and the straight line $y = 2x - 2$. Sketch $R$ and calculate its area.
14. (10 points) Let $T$ denote the triangular region with vertices at $(0, 0), (2, 1)$ and $(3, 1)$. Sketch $T$ and use the method of disks to compute the volume of the solid obtained by rotating $T$ about the $y$-axis.
15. (10 points) Let \( R \) denote the region enclosed (in the first quadrant) by the \( x \)-axis, the line \( x = 1 \), and the curve \( y = x^3 \). Sketch \( R \) and use the method of \textit{cylindrical shells} to calculate the volume of the solid obtained by rotating \( R \) about the line \( x = 1 \).
16. (6 points) Let $f$ be a function such that $f''$ is continuous in the interval $[0, \pi]$. Given that

$$f(0) = 1, \quad f(\pi) = -1, \quad \text{and} \quad \int_0^\pi f(x) \sin x \, dx = 3,$$

evaluate

$$\int_0^\pi f''(x) \sin x \, dx.$$
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