1. **E** Integrate by parts with \( u = x^2, \ dv = e^x \ dx. \) Then \( \int x^2 e^x \ dx = x^2 e^x - \int 2x e^x \ dx. \) Integrate by parts again with \( u = 2x, \ dv = e^x \ dx \) yields \( x^2 e^x - 2xe^x + \int 2e^x \ dx = x^2 e^x - 2xe^x + 2e^x + C. \)

2. **E** The force (weight) of the rope and weight after it has been pulled up \( x \) feet is \( F(x) = 200 - 2x. \) Therefore, the work done is given by \( W = \int_0^2 (200 - 2x) \ dx = 200x - x^2 \bigg|_0^2 = 4000 - 400 = 3600 \) ft-lbs.

3. **B** Let \( u = x^2 + 1. \) Then \( du = 2x \ dx. \) When \( x = 0, u = 1, \) and when \( x = 1, u = 2. \) Substituting into the integral yields 
\[
\int_1^2 \frac{1}{2} u^{-1/2} \ du = u^{1/2} \bigg|_{1}^{2} = \sqrt{2} - 1.
\]

4. **C** Using the double-angle identity for \( \sin(2x) \) yields \( \int_0^{\pi/2} 2 \sin(x) \cos^2(x) \ dx. \) Let \( u = \cos(x). \) Then \( du = -\sin(x) \ dx. \) When \( x = 0 u = 1, \) and when \( x = \frac{\pi}{2} u = 0. \) Substituting into the integral yields 
\[
\int_0^1 2u^2 \ du = \int_0^1 2u^2 \ du = 2 \left. \frac{u^3}{3} \right|_0^1 = \frac{2}{3}.
\]

5. **D** Partition along the \( y \)-axis. The slices are square prisms \( 2x \times 2x \times dy = 4x^2 \ dy = 4y \ dy. \) Therefore \( V = \int_0^1 4y \ dy = 2y^2 \bigg|_0^1 = 2. \)

6. **B** Let \( u = 10 + x. \) Then \( du = dx. \) When \( x = -1, u = 9, \) and when \( x = 6, u = 16. \) The integral is therefore equivalent to \( \int_9^{16} \frac{x}{\sqrt{u}} \ du. \) Since \( u = 10 + x, \) \( x = u - 10, \) so this integral is equivalent to 
\[
\int_9^{16} \frac{u - 10}{\sqrt{u}} \ du = \int_9^{16} (u^{1/2} - 10u^{-1/2}) \ du.
\]

7. **A** The sketch of the region is shown below. Use a partition of the \( y \)-axis, so the functions are \( x = y + 1, \) and \( x = y^2 - 1. \) These functions intersect when \( y + 1 = y^2 - 1, \) \( y^2 - y - 2 = 0, \) \( (y - 2)(y + 1) = 0, \) or \( y = 2, -1. \) The area is therefore \( \int_{-1}^{2} (y + 1) - (y^2 - 1) \ dy. \)

8. **A** Create cylindrical shells centered around \( x = 2. \) The volume of a shell is \( V = 2\pi rh \ dv = 2\pi(2-x)(x-x^2) \ dx. \) Since the curves intersect at \( x = 0 \) and \( x = 1, \) the volume of the solid is \( V = 2\pi \int_0^1 (2-x)(x-x^2) \ dx. \)

9. **B** Integrate by parts with \( u = \ln x, \ dv = x^3 \ dx. \) Then \( du = \frac{1}{x} \ dx \) and \( v = \frac{1}{4} x^4. \) The integral is equivalent to 
\[
\int \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^4 \left( \frac{1}{x} \right) \ dx = \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^3 \ dx.
\]

10. **D** \( \sec^4 x = \sec^2 x \cdot \sec^2 x = (\tan^2 x + 1) \sec^2 x. \) Let \( u = \tan x. \) Then \( du = \sec^2 x \ dx. \) When \( x = 0, u = 0, \) and when \( x = \frac{\pi}{4}, u = 1. \) Substituting into the integral yields 
\[
\int_0^1 (u^2 + 1) \ du = \left. \frac{1}{3} u^3 + u \right|_0^1 = \frac{4}{3}.
\]
11. The graph is shown below. When \( x = 8 \), \( y = 16 \) and \( f'(x) = 4 \), so the equation of the tangent line is \( y = 16 + 4(x - 8) = 4x - 16 \). It is easiest to partition the \( y \)-axis, so the functions become \( x = 2\sqrt{y} \) and \( x = \frac{1}{4}(y + 16) \). The area is given by \( \int_0^{16} \left( \frac{1}{4}y + 4 \right) - 2\sqrt{y} \, dy \). Integrating yields \( \frac{1}{8}y^2 + 4y - \frac{4}{3}y^{3/2} \bigg|_0^{16} = \left( 32 + 64 - \frac{256}{3} \right) = \frac{32}{3} \).

12. 

(a) Let \( u = \tan^{-1}x \) and \( dv = x \, dx \). Then \( du = \frac{1}{x^2 + 1} \, dx \) and \( v = \frac{1}{2}x^2 \). Then \( \int x \tan^{-1}x \, dx \)

\[
= \frac{1}{2}x^2 \tan^{-1}x - \int \frac{1}{2}x^2 \frac{1}{x^2 + 1} \, dx,
\]

\[
= \frac{1}{2}x^2 \tan^{-1}x - \frac{1}{2} \int \frac{1}{x^2 + 1} \, dx = \frac{1}{2}x^2 \tan^{-1}x - \frac{1}{2} \left( \frac{1}{x^2 + 1} \right) + C = \frac{1}{2}x^2 \tan^{-1}x - \frac{1}{2}(x - \tan^{-1}x) + C.
\]

(b) \( \int \cos^2x \tan^3x \, dx = \int \frac{\sin^3x}{\cos x} \, dx \)

\[
= \int \frac{(1 - \cos^2x) \sin x}{\cos x} \, dx \quad \text{Let} \quad u = \cos x \quad \text{so the integral is equivalent to}
\]

\[- \int \frac{1 - u^2}{u} \, du = - \int \left( \frac{1}{u} - u \right) \, du = - \ln|u| + \frac{1}{2}u^2 + C = - \ln|\cos x| + \frac{1}{2}\cos^2x + C.
\]

13. A cross-sectional slice a distance \( y \) from the bottom is cylindrical with radius \( x \) and height \( dy \). From similar triangles, \( \frac{2}{3} = \frac{x}{y} \), so \( x = \frac{2}{3}y \). The force (weight) acting on the slice is \( F = \rho g \pi \left( \frac{2}{3}y \right)^2 \, dy \). The distance required to move the slice out of the tank is \( 4 - y \) (including the spout), so the work required to pump the water out of the tank is \( W = \int_0^3 \rho g \pi \left( \frac{4}{3}y^2(4 - y) \right) \, dy \)

\[
= \frac{4\rho g \pi}{9} \int_0^3 (4y^2 - y^3) \, dy = \frac{4\rho g \pi}{9} \left( \frac{4y^3 - \frac{4}{3}y^4}{3} \right) \bigg|_0^3 = \frac{4\rho g \pi}{9} \left( 36 - \frac{81}{4} \right) = 7\rho g \pi \text{ ft-lbs.}
\]

14. Slice perpendicular to the \( x \)-axis. The slices are washers with outer radius \( \sec x \), inner radius \( 2\sin x \), and height \( dx \). Therefore, the volume of the solid is \( V = \pi \int_0^{\pi/4} (\sec^2x - 4\sin^2x) \, dx \)

\[
= \pi \int_0^{\pi/4} \left( \sec^2x - 2(1 - \cos(2x)) \right) \, dx = \pi \left( \tan(x) - 2x + \sin(2x) \right) \bigg|_0^{\pi/4} = \pi \left( 1 - \frac{\pi}{2} + 1 \right) = 2\pi - \frac{\pi^2}{2}.
\]

15. The average value is found by calculating \( \frac{1}{b - 0} \int_0^b (3x^2 + 4x - 7) \, dx = \frac{1}{b} \left( x^3 + 2x^2 - 7x \right) \bigg|_0^b = b^3 + 2b - 7 \). Set this equal to 8 and solve: \( b^2 + 2b - 7 = 8 \), \( b^2 + 2b - 15 = 0 \), \( (b + 5)(b - 3) = 0 \), so \( b = -5 \) or \( b = 3 \). Since \( b > 0 \), our only solution is \( b = 3 \).