1. A As $n \to \infty$, $e^{-n} \to 0$, so $a_n \to \frac{1}{5-0} = \frac{1}{5}$.

2. B Since $\sin^2 x \leq 1$, $\sin^2 x \leq \frac{1}{x^2}$. The Type-1 improper integral $\int_{1}^{\infty} \frac{1}{x^p} \, dx$ converges if and only if $p > 1$, so $\int_{1}^{\infty} \frac{\sin^2 x}{x^2} \, dx$ converges by comparison with $\int_{1}^{\infty} \frac{1}{x^2} \, dx$.

3. E $|a_n| = \frac{2n^2 + 2}{3n^2 + 1} \to \frac{2}{3}$, so the terms of the sequence alternate between approaching $\frac{2}{3}$ (even-numbered terms) and approaching $-\frac{2}{3}$ (odd-numbered terms), which means the sequence diverges.

4. D The Test for Divergence fails since $\frac{n}{n^3 - 5} \to 0$. Compare with $\sum_{n=2}^{\infty} \frac{1}{n^2}$, which is convergent by the P-test. Since $\frac{n}{n^3 - 5} > \frac{1}{n^2}$, the Comparison Test fails. Using the Limit Comparison Test, we see that
$$\lim_{n \to \infty} \frac{n}{n^3 - 5} = 0,$$
which means the given series is convergent.

5. D (a), (c), and (d) are P-series with $p = 1, \frac{3}{3}, 2$, respectively, so only (d) is convergent. Since $\int_{2}^{\infty} \frac{1}{x \ln x} \, dx = \lim_{a \to \infty} \ln(\ln x)|_{2}^{a} = \infty$, (b) is divergent by the Integral Test.

6. E The denominator contains a repeating linear factor and an irreducible quadratic factor, so the form of the partial fraction decomposition is $\frac{A}{x + 1} + \frac{B}{(x + 1)^2} + \frac{Cx + D}{x^2 + 2x + 3}$.

7. E The function is unbounded at $x = 0$, so we rewrite the integral as $\int_{-1}^{0} \frac{1}{x^2} \, dx + \int_{0}^{3} \frac{1}{x^2} \, dx$.

The Type-2 improper integral $\int_{0}^{a} \frac{1}{x^p} \, dx$ converges if and only if $p < 1$, so both integrals diverge.

8. C If $S$ is the surface area, $S = \int_{a}^{b} 2\pi r \, ds$.

We choose to integrate with respect to $x$, so $r = y = e^{2x}$. $\frac{dy}{dx} = 2e^{2x}$, so $ds = \sqrt{1 + (2e^{2x})^2} = \sqrt{1 + 4e^{4x}}$. Therefore, $S = \int_{0}^{1} 2\pi e^{2x} \sqrt{1 + 4e^{4x}} \, dx$.

9. A Let $s$ be the sum of the series. Then $s = \lim_{n \to \infty} s_n = 2$, so statement (I) is true. This means the series is convergent, which makes (II) false. Since the series is convergent, $a_n \to 0$ by the Test for Divergence, so (III) is true.

10. B Let $x = 2\sin \theta$. Then $dx = 2 \cos \theta \, d\theta$.

When $x = \sqrt{3}$, $\theta = \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) = \frac{\pi}{3}$.

When $x = 2$, $\theta = \sin^{-1} \left( \frac{2}{2} \right) = \frac{\pi}{2}$. Then
$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{4 - x^2} \, dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sqrt{4 - 4 \sin^2 \theta} \,(2 \cos \theta \, d\theta)$$
$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \,(2 \cos \theta)(2 \cos \theta) \, d\theta = 4 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta.$$
11. Let \( x = 2 \sec \theta \). Then \( dx = 2 \sec \theta \tan \theta \, d\theta \).

Substituting into the integral yields

\[
\int \frac{2 \sec \theta \tan \theta}{4 \sec^2 \theta \sqrt{4 \sec^2 \theta - 4}} \, d\theta = \int \frac{2 \sec \theta \tan \theta}{4 \sec^2 \theta (2 \tan \theta)} \, d\theta
\]

Using the reference triangle below,

\[
\sin \theta = \frac{\sqrt{x^2 - 4}}{x},
\]

so the integral is

\[
= \frac{1}{4} \int_1^2 \frac{1}{\cos \theta} \, d\theta = \frac{1}{4} \int_0^{\pi/2} \frac{1}{\sin \theta} \, d\theta = \frac{1}{4} \sin \theta + C.
\]

Using both series are convergent.

Therefore, the sum is given by

\[
\sum_{n=0}^{\infty} \frac{2^n}{10^n} = \sum_{n=0}^{\infty} \frac{2^n}{10^n},
\]

assuming both series are convergent. The first is a geometric series with \( a = 2, r = \frac{1}{10} \), and the second is a geometric series with \( a = 1, r = \frac{2}{10} \), so both series are convergent and the total sum is

\[
\frac{2}{1 - \frac{1}{10}} + \frac{1}{1 - \frac{2}{10}} = \frac{20}{9} + \frac{5}{4} = \frac{125}{36}.
\]

(b) Using partial fractions, we find that

\[
\frac{2}{(n+1)(n+3)} = \frac{1}{n+1} - \frac{1}{n+3}.
\]

Therefore, the Nth partial sum of the series is given by

\[
s_N = \left(1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{N-1} - \frac{1}{N+1}\right) + \left(\frac{1}{N+1} - \frac{1}{N+3}\right) = 1 + \frac{1}{2} - \frac{1}{N+2} - \frac{1}{N+3} + \cdots + \frac{1}{N}\]

as a telescoping series. Since

\[
s = \lim_{N \to \infty} s_N = 1 + \frac{1}{2},
\]

the series converges to \( \frac{3}{2} \).

13. (a) \( \frac{dx}{dt} = -\sin t + \sin t + t \cos t = t \cos t \).

\( \frac{dy}{dt} = -\cos t - \cos t + t \sin t = t \sin t \).

Therefore, the length of the curve is given by

\[
\int_0^{\pi/2} \sqrt{(t \cos t)^2 + (t \sin t)^2} \, dt = \int_0^{\pi/2} t \, dt
\]

\[
= \frac{1}{2} t^2 \bigg|_0^{\pi/2} = \frac{\pi^2}{8}.
\]

Therefore, the integral converges and therefore \( \sum_{n=0}^{\infty} \frac{2^n}{10^n} \) converges to \( \frac{125}{36} \).

(b) Using partial fractions, we find that

\[
\frac{1}{4 \sec \theta \sqrt{4 \sec^2 \theta - 4}} = \frac{1}{4} \int_0^{\pi/2} \frac{1}{\sin \theta} \, d\theta = \frac{1}{4} \sin \theta + C.
\]

14. \( \frac{3x^2 - 4x + 11}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4} \). Eliminating the fractions yields

\[
3x^2 - 4x + 11 = A(x^2 + 4) + (Bx + C)(x-1).
\]

If \( x = 1, \ 10 = 5A, \) so \( A = 2 \). Expanding the right-hand side yields

\[
3x^2 - 4x + 11 = 2x^2 + 8 + Bx^2 - Bx + Cx - C.
\]

From the \( x^2 \) coefficients, we must have \( B = 1 \), and from the constants, we must have \( C = -3 \). Therefore, the given integral is equivalent to

\[
\int \left( \frac{2}{x-1} + \frac{x-3}{x^2+4} \right) \, dx
\]

\[
= \int \left( \frac{2}{x-1} + \frac{x}{x^2+4} - \frac{3}{x^2+4} \right) \, dx
\]

\[
= 2 \ln |x-1| + \frac{1}{2} \ln |x^2+4| - \frac{3}{2} \tan^{-1} \left( \frac{x}{2} \right) + C.
\]

15. (a) Let \( f(x) = 3x^2 e^{-x^3} \). \( f \) is continuous, positive, and decreasing \( (f'(x) = 3x^2 e^{-x^3} (2 - 3x^3) < 0) \), so we can apply the Integral Test.

\[
\int_1^{\infty} 3x^2 e^{-x^3} \, dx = \lim_{a \to \infty} -e^{-x^3} \bigg|_1^{a} = e^{-1},
\]

so the integral converges and therefore the given series is convergent by the Integral Test.

(b) By the remainder theorem,

\[
s - s_3 \leq \int_3^{\infty} 3x^2 e^{-x^3} \, dx \leq s - s_3 \leq \lim_{a \to \infty} -e^{-x^3} \bigg|_3^{a} = e^{-27}.
\]