1. The sequence whose terms are \( a_n = \frac{n^2 - 1}{n^2} \)
   
   (a) increases and converges to 1.
   (b) decreases and converges to 1.
   (c) increases and converges to 0.
   (d) decreases and converges to 0.
   (e) diverges.

2. By substituting \( x = 3 \tan \theta \), the integral \( \int_0^3 x^2 \sqrt{x^2 + 9} \, dx \) becomes
   
   (a) \( \int_0^{\pi/4} 81 \tan^2 \theta \sec^2 \theta \, d\theta \)
   (b) \( \int_0^{\pi/4} 81 \tan^3 \theta \sec^2 \theta \, d\theta \)
   (c) \( \int_0^{\pi/4} 27 \tan^2 \theta \sec \theta \, d\theta \)
   (d) \( \int_0^{\pi/4} 81 \tan^2 \theta \sec^3 \theta \, d\theta \)
   (e) \( \int_0^3 27 \tan^2 \theta \sec^3 \theta \, d\theta \)

3. \( \sum_{n=0}^{\infty} \frac{(-1)^n + 2^n}{6^n} = \)
   
   (a) \( \frac{5}{14} \)
   (b) \( \frac{27}{10} \)
   (c) \( \frac{3}{10} \)
   (d) \( \frac{33}{14} \)
   (e) \( \frac{1}{3} \)
4. Which of the following series diverges by the Test for Divergence?

(a) \( \sum_{n=1}^{\infty} \sin \left( \frac{\pi}{2} - \frac{1}{n} \right) \)

(b) \( \sum_{n=1}^{\infty} \frac{\ln n}{n} \)

(c) \( \sum_{n=1}^{\infty} \sin \left( \frac{1}{n} \right) \)

(d) \( \sum_{n=1}^{\infty} \frac{n}{n!} \)

(e) The Test for Divergence is inconclusive for all of the above series.

5. Find the length of the curve \( x = t^2, \ y = t^3 \), for \( 0 \leq t \leq 1 \).

(a) \( \frac{2\pi}{27} (13\sqrt{13} - 1) \)

(b) \( \frac{1}{27} \)

(c) \( \frac{1}{27} (13\sqrt{13} - 1) \)

(d) \( \frac{2\pi}{27} (13\sqrt{13} - 8) \)

(e) \( \frac{1}{27} (13\sqrt{13} - 8) \)

6. Find the surface area obtained by rotating the curve \( x = \cos(2t), \ y = \sin(2t) \), for \( 0 \leq t \leq \frac{\pi}{4} \), about the x-axis.

(a) \( \frac{\pi}{2} \)

(b) \( \pi \)

(c) \( \frac{\pi}{4} \)

(d) \( 4\pi \)

(e) \( 2\pi \)

7. Find the sum of the geometric series \( S = \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \ldots \)

(a) \( S = 2 \)

(b) \( S = 3 \)

(c) \( S = \frac{4}{3} \)

(d) \( S = \frac{4}{15} \)

(e) \( S = \frac{2}{3} \)
8. Which of the following statements is true regarding the improper integral \( \int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} \)?

(a) The integral converges to 0.
(b) The integral converges because \( \int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} < \int_1^{\infty} \frac{dx}{e^x} \) and \( \int_1^{\infty} \frac{dx}{e^x} \) converges.
(c) The integral diverges because \( \int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} > \int_1^{\infty} \frac{dx}{\sqrt{x}} \) and \( \int_1^{\infty} \frac{dx}{\sqrt{x}} \) diverges.
(d) The integral diverges because \( \int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} > \int_1^{\infty} \frac{dx}{e^x} \) and \( \int_1^{\infty} \frac{dx}{e^x} \) diverges.
(e) The integral converges because \( \int_1^{\infty} \frac{dx}{e^x + \sqrt{x}} < \int_1^{\infty} \frac{dx}{\sqrt{x}} \) and \( \int_1^{\infty} \frac{dx}{\sqrt{x}} \) converges.

9. The recursive sequence defined by \( a_1 = 2, \ a_{n+1} = 5 - \frac{4}{a_n} \) converges. Find the limit.

(a) 4
(b) 5
(c) 1
(d) \( \frac{5}{2} \)
(e) 2

10. \( \int \frac{dx}{x^2(x-1)} \) =

(a) \( \ln |x - 1| - \frac{1}{x} + C \)
(b) \( \ln |x| - \frac{1}{x} - \ln |x - 1| + C \)
(c) \( -\ln |x| + \frac{1}{x} + \ln |x - 1| + C \)
(d) \( \ln |x - 1| + \frac{1}{x} + C \)
(e) \( \ln |x^2(x - 1)| + C \)
11. Compute \( \int_{-1}^{\infty} \frac{dx}{1 + x^2} \).

(a) \( \frac{\pi}{4} \)
(b) \( \frac{\pi}{2} \)
(c) \( \frac{3\pi}{4} \)
(d) \( \infty \)
(e) 0

12. Which of the following integrals gives the surface area obtained by rotating the curve \( y = e^{-4x} \), for \( 0 \leq x \leq 1 \), about the \( y \)-axis?

(a) \( \int_{0}^{1} 2\pi x \sqrt{1 + 16e^{-8x}} \, dx \)
(b) \( \int_{0}^{1} 2\pi e^{-4x} \sqrt{1 + 16e^{-8x}} \, dx \)
(c) \( \int_{1}^{e} 2\pi y \sqrt{1 + \frac{1}{16y^2}} \, dy \)
(d) \( \int_{0}^{1} \frac{\pi}{2} \sqrt{16y^2 + 1} \, dy \)
(e) \( \int_{0}^{1} \frac{\pi \ln y}{8} \sqrt{16y^2 + 1} \, dy \)

13. The improper integral \( \int_{1}^{e} \frac{dx}{x \ln x} \)

(a) diverges to \(-\infty\).
(b) diverges to \(\infty\).
(c) converges to \(-1\)
(d) converges to 1.
(e) converges to \(\frac{1}{e} - 1\).
PART II: WORK OUT (52 points total)

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

14. (10 pts) Integrate \( \int \sqrt{16 - 9x^2} \, dx \).

15. (8 pts) Find the sum of the series: \( S = \sum_{n=1}^{\infty} \left( \cos \frac{\pi}{n} - \cos \frac{\pi}{n + 1} \right) \).
16. (10 pts) Integrate \( \int \frac{4x^2 - 1}{(x^2 + 1)(x - 2)} \, dx \).
17. If the $n$th partial sum of the series $\sum_{n=1}^{\infty} a_n$ is given by $s_n = \frac{2n + 1}{n}$, 

(i) (5 pts) Find $a_{10}$.

(ii) (5 pts) Find the sum of the series $S = \sum_{n=1}^{\infty} a_n$.

18. (10 pts) Find the surface area obtained by rotating the curve $y = \frac{x^2}{4} - \frac{1}{2} \ln x$, for $1 \leq x \leq 2$, about the $y$-axis.
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