MATH 152, FALL 2012
COMMON EXAM III - VERSION B

Print name (LAST, First): ________________________________ SECTION #: __________

INSTRUCTOR: ________________________________ SEAT #: __________

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"An Aggie does not lie, cheat, or steal, or tolerate those who do." By signing below, you indicate that all work is your own and that you have neither given nor received help from any external sources.

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PART I-MULTIPLE CHOICE
The use of any electronic device is prohibited. Mark the correct choice on your ScanTron using a No. 2 pencil. For your own records, also record your choices on your exam! Be sure to write your name, section and version letter of the exam on the ScanTron form. Each problem is worth 3 points.

NOTE: The following formulas may or may not be useful on this exam:
(I) If \(|f^{(n+1)}(x)| \leq M\) on the interval \([a, b]\), then \(|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}\) for \(a \leq x \leq b\).
(II) \(s - s_n \leq \int_n^\infty f(x) \, dx\)

1. Find the center and radius of the sphere \((x + 2)^2 + (y - 3)^2 + z^2 - 8z + 7 = 0\)

(a) center \((2, -3, -4)\), radius 3
(b) None of these
(c) center \((-2, 3, 8)\), radius 7
(d) center \((-2, 3, 4)\), radius 3
(e) center \((-2, 3, 4)\), radius 9

2. Given the radius of convergence of the series \(\sum_{n=0}^{\infty} \frac{(-1)^n(x - 3)^n}{4n + 1}\) is 1, what is the interval of convergence?

(a) \([2, 4]\)
(b) \((2, 4]\)
(c) \((-1, 1)\)
(d) None of these
(e) \([-1, 1]\)
3. The series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \) converges to \( s \). Based on the Alternating Series Estimation Theorem, which statement is true?

(a) \( |s - s_8| < \frac{1}{9} \)
(b) \( |s - s_8| < \frac{1}{8} \)
(c) None of these
(d) \( |s - s_8| < \frac{1}{81} \)
(e) \( |s - s_8| < \frac{1}{64} \)

4. Describe the surface in \( \mathbb{R}^3 \) represented by the equation \( x^2 + y^2 = 1 \).

(a) A sphere
(b) A cylinder
(c) A circle
(d) A plane
(e) None of these

5. What is the radius of convergence of \( \sum_{n=0}^{\infty} \frac{n!x^n}{(2012)^n} \)?

(a) \( \infty \)
(b) None of these
(c) \( \frac{1}{2012} \)
(d) 0
(e) 2012

6. Given the points \( A(1, 1, -1) \) and \( B(2, 0, 1) \), which vector is orthogonal to the vector in the direction from \( A \) to \( B \)?

(a) \( (1, 1, 1) \)
(b) \( (1, 0, -2) \)
(c) \( (2, 2, 0) \)
(d) \( (1, -1, 2) \)
(e) None of these
7. Find the sum: \( \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!} \).

(a) 0
(b) \( e^{-\pi^2} \)
(c) \( \frac{1}{1 + \pi^2} \)
(d) None of these
(e) \(-1\)

8. Given the power series for \( \frac{1}{1-x} \) is \( \sum_{n=0}^{\infty} x^n \) for \(-1 < x < 1\), which of the following is a power series for \( f(x) = \frac{1}{(1-x)^2} \)?

(a) \( \sum_{n=1}^{\infty} (-1)^n n x^{n-1} \)
(b) \( \sum_{n=0}^{\infty} x^{2n} \)
(c) \( \sum_{n=1}^{\infty} n x^{n-1} \)
(d) \( \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} \)
(e) None of these

9. Which statement is true regarding the series \( \sum_{n=1}^{\infty} (-1)^n \frac{4}{\sqrt{n^2 + 4} + n} \):

(a) None of the other statements is true.
(b) The series is absolutely convergent by the Test for Divergence.
(c) The series is absolutely convergent by the Ratio Test.
(d) The series is divergent.
(e) The series is convergent, but not absolutely convergent.

10. Which statement is true regarding the convergence or divergence of \( \sum_{n=1}^{\infty} \frac{1 + \cos^2 n}{n^2} \)?

(a) The series converges by the Comparison Test with \( \sum_{n=1}^{\infty} \frac{2}{n^2} \).
(b) The series converges by the Limit Comparison Test with \( \sum_{n=1}^{\infty} \frac{2}{n^2} \).
(c) The series diverges by the Comparison Test with \( \sum_{n=1}^{\infty} \frac{1}{n} \).
(d) The series diverges by the Limit Comparison Test with \( \sum_{n=1}^{\infty} \frac{1}{n} \).
(e) None of the other statements is true.
11. For which series below is the Ratio Test inconclusive?

(a) \[ \sum_{n=0}^{\infty} \frac{3^n}{5^{n-1}} \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-1)^n}{4n^2 + 3n + 3} \]

(c) \[ \sum_{n=0}^{\infty} \frac{1}{2^n} \]

(d) \[ \sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n!} \]

(e) None of these

12. Which of the following is the scalar projection of the vector \( \mathbf{i} + \mathbf{j} + 2\mathbf{k} \) onto the vector \(-2\mathbf{i} + 3\mathbf{j} + \mathbf{k}\)?

(a) \[ -\frac{3}{7} \mathbf{i} + \frac{9}{14} \mathbf{j} + \frac{3}{14} \mathbf{k} \]

(b) \[ \frac{1}{2} \mathbf{i} + \frac{1}{2} \mathbf{j} + \mathbf{k} \]

(c) \[ \frac{3}{\sqrt{14}} \]

(d) None of these

(e) \[ \frac{3}{\sqrt{6}} \]

13. Given the Maclaurin Series for a function \( f \) is defined by \( f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n^{2n}} \), what is \( f^{(10)}(0) \) (the tenth derivative of \( f \) at \( x = 0 \))? 

(a) \[ \frac{1}{10 \cdot 2^{10}} \]

(b) \[ \frac{9!}{2^{10}} \]

(c) None of these

(d) \[ -\frac{9!}{2^{10}} \]

(e) \[ -\frac{1}{10 \cdot 2^{10}} \]

14. Which of the following is the 3rd degree Taylor Polynomial for \( f(x) = x + \sin(2x) \) centered at \( a = 0 \)?

(a) \( T_3(x) = 2x - \frac{1}{3} x^3 \)

(b) None of these

(c) \( T_3(x) = 3x - \frac{4}{3} x^3 \)

(d) \( T_3(x) = 3x - 8x^3 \)

(e) \( T_3(x) = 2x - 2x^3 \)
15. Which of the following is a unit vector in the direction from the point \( P(3, -3, 3) \) to the point \( Q(2, -1, 2) \)?

(a) \(-\frac{1}{\sqrt{6}}i + \frac{2}{\sqrt{6}}j - \frac{1}{\sqrt{6}}k\)
(b) \(-\frac{5}{\sqrt{66}}i + \frac{4}{\sqrt{66}}j - \frac{5}{\sqrt{66}}k\)
(c) \(\frac{5}{\sqrt{66}}i - \frac{4}{\sqrt{66}}j + \frac{5}{\sqrt{66}}k\)
(d) \(\frac{1}{\sqrt{6}}i - \frac{2}{\sqrt{6}}j + \frac{1}{\sqrt{6}}k\)
(e) None of these

**PART II WORK OUT**

**Directions:** Present your solutions in the space provided. *Show all your work* neatly and concisely and **BOX YOUR FINAL ANSWER.** You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

**REMEMBER:** The following formulas may or may not be useful on this exam:

(I) If \(|f^{(n+1)}(x)| \leq M\) on the interval \([a, b]\), then \(|R_n(x)| \leq \frac{M}{(n+1)!}|x - a|^{n+1}\) for \(a \leq x \leq b\).

(II) \(s - s_n \leq \int_n^\infty f(x) \, dx\)

16. (5 points) Find the Taylor Series for \(f(x) = e^{-x}\) centered at \(a = 2\).
(a) (6 points) Find a power series representation for \( f(x) = \frac{1}{3 + 6x} \) about \( a = 0 \).

(b) (2 points) What is the radius of convergence of the series in part (a)?

(c) (4 points) Use your series in part (a) to find the Maclaurin series for \( \ln(3 + 6x) \).
18. (8 points) Find the radius and interval of convergence of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n(x - 3)^n}{\sqrt[3]{n + 2}} \). Be sure to test the endpoints for convergence.
19. (6 points each) Given \( f(x) = \cos x \):

(a) Find the second degree Taylor polynomial \( T_2(x) \) for \( f \) centered at \( a = \frac{\pi}{3} \).

(b) Use Taylor's Inequality to estimate the accuracy of the approximation \( f(x) \approx T_2(x) \) when \( 0 \leq x \leq \frac{\pi}{2} \).
20. Let \( b_n = ne^{-n^2} \).

(a) (5 pts) Show that the series \( \sum_{n=1}^{\infty} (-1)^{n-1}b_n \) converges. Clearly explain your reasoning.

(b) (3 pts) According to Matlab, the third partial sum of the series in part (a) is \( s_3 \approx 0.3316183928 \). Find an upper bound on the error \( |s - s_3| \).

(c) (5 pts) Show that the series \( \sum_{n=1}^{\infty} b_n \) converges. Clearly explain your reasoning.

(d) (5 pts) According to Matlab, the third partial sum of the series in part (c) is \( s_3 \approx 0.404880948 \). Find an upper bound on the error \( |s - s_3| \).
MATH 152 Exam III

Print name (LAST, First):

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SECTION #: ______