1. D \( |a_n| = \frac{n^2}{7n^3 + 1} \). As \( n \to \infty \), \( |a_n| \to 0 \), so \( a_n \to 0 \).

2. A \( \sum_{n=1}^{\infty} \frac{6n + 3}{n + 1} \) is divergent by the Test for Divergence. \( \sum_{n=1}^{\infty} \frac{6n + 3}{n(n + 1)} \) is divergent by the Limit Comparison Test with \( \sum_{n=1}^{\infty} \frac{6}{n} \). \( \sum_{n=1}^{\infty} \frac{6n + 3}{n^2(n + 1)} \) is convergent by the Comparison (or Limit Comparison) Test with \( \sum_{n=1}^{\infty} \frac{6}{n^2} \).

3. B \( S = \int 2\pi R \, ds \). Since the curve is rotated about the \( x \)-axis, \( R = y \). Choosing to evaluate arclength with respect to \( x \) (meaning \( R = \sqrt{r^2 - x^2} \)), \( ds = \sqrt{1 + \left( \frac{1}{2} (r^2 - x^2)^{-1/2} (-2x) \right)^2} \, dx = \sqrt{1 + \frac{x^2}{r^2 - x^2}} \, dx \). Therefore, \( S = \int_{-r}^{r} 2\pi \sqrt{r^2 - x^2} \sqrt{1 + \frac{x^2}{r^2 - x^2}} \, dx \).

4. A Since \( s_n \to \frac{1}{2} \), the series converges to \( \frac{1}{2} \).

5. A By \#4 the series is convergent. The Test for Divergence therefore says that \( a_n \to 0 \).

6. E \( s_n = \sum_{i=1}^{n} (e^{1/i} - e^{1/(i+1)}) = (e^{1/1} - e^{1/2}) + (e^{1/2} - e^{1/3}) + \ldots + (e^{1/n} - e^{1/(n+1)}) \). Cancelling terms (telescoping) yields \( s_n = e - e^{1/(n+1)} \). As \( n \to \infty \), \( \frac{1}{n+1} \to 0 \), so the series converges to \( e - e^0 = e - 1 \).

7. D The denominator contains a repeating linear factor and an irreducible quadratic factor, so the form of the partial fraction decomposition is \( \frac{A}{x - 4} + \frac{B}{(x - 4)^2} + \frac{C}{x^2 + 4x + 16} \).

8. B Complete the square to yield \( \int \frac{1}{\sqrt{(x - 4)^2 + 4}} \, dx \).
Then the desired substitution is \( x - 4 = 2 \tan \theta \).

9. E The series is geometric with \( a = 3 \) and \( r = \frac{2}{3} \).
Since \( |r| < 1 \), the series converges to \( \frac{3}{1 - \frac{2}{3}} = 9 \).

10. A Let \( u = -x^2 \). Then \( du = -2x \, dx \), so \( \int_{0}^{\infty} 6xe^{-x^2} \, dx = \lim_{a \to \infty} -3e^{-x^2} \bigg|_{0}^{a} = \lim_{a \to \infty} -3e^{-a^2} + 3 = 3 \).

11. C \( a_n = \ln \left( \frac{3n^2 + 1}{n^2 + 1} \right) \). As \( n \to \infty \), \( \frac{3n^2 + 1}{n^2 + 1} \to 3 \), so \( a_n \to \ln 3 \).

12. B \( a_1 = 3, a_2 = \frac{3}{3 - 1} = \frac{3}{2}, a_3 = \frac{3/2}{3/2 - 1} = 3, a_4 = \frac{3}{2} \), so the sequence oscillates between 3 and \( \frac{3}{2} \) and therefore diverges.

13. D \( \int_{0}^{1} \frac{1}{4x - 2} \, dx = \int_{0}^{1/2} \frac{1}{4x - 2} \, dx + \int_{1/2}^{1} \frac{1}{4x - 2} \, dx = \lim_{a \to 1/2^-} \int_{0}^{a} \frac{1}{4x - 2} \, dx + \lim_{a \to 1/2^+} \int_{a}^{1} \frac{1}{4x - 2} \, dx = \frac{1}{4} (\ln|4a - 2| - \ln|2|) + \frac{1}{4} (\ln|10| - \ln|4a - 2|) \), which diverges since \( \lim_{a \to 1/2} \ln|4a - 2| = -\infty \).

14. D The debris hits the ground when \( y = 0 \), or \( 0 = \frac{9}{2} - \frac{1}{8} x^2 \), \( x^2 = 36 \), or \( x = 6 \).
\( \frac{dy}{dx} = -\frac{1}{4} x \), so the length of the curve is \( s = \int_{0}^{6} \sqrt{1 + \left( \frac{-1}{4} x \right)^2} \, dx = \int_{0}^{6} \sqrt{1 + \frac{x^2}{16}} \, dx \).

15. A Since the first term is \( a_1 \), the numerator of each term is \( n \) and the denominator (with slope = 2 and equal to 1 when \( n = 1 \)) is \( 2n - 1 \). Therefore, \( a_n = \frac{n}{2n - 1} \).
16. \( ds = \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \cdot \frac{dx}{dt} = e^t - e^{-t}, \) and \( \frac{dy}{dt} = 2. \) Then \( s = \int_0^2 \sqrt{(e^t - e^{-t})^2 + 2^2} dt = \int_0^2 \sqrt{2e^{2t} - 2 + e^{-2t} + 4} dt = \int_0^2 \sqrt{e^{2t} + 2 + e^{-2t}} dt = \int_0^2 (e^t + e^{-t}) dt = e^t - e^{-t}|_0^2 = (e^2 - e^{-2}) - (1 - 1).

17. \( S = \int 2\pi r \, ds. \) Since the curve is rotating about the \( y \)-axis, \( r = x. \) It is easier to integrate with respect to \( y, \) so \( r = y^3 \) and \( ds = \sqrt{\left( \frac{dx}{dy} \right)^2 + 1} dy = \sqrt{(3y^2)^2 + 1} dy = \sqrt{9y^4 + 1} dy. \) Therefore, \( S = \int_1^3 2\pi y^3 \sqrt{9y^4 + 1} dy. \) Let \( u = 9y^4 + 1. \) Then \( du = 36y^3 \, dy, \) so \( S = 2\pi \left( \frac{1}{36} \right) \left( \frac{2}{3} \right) (9y^4 + 1)^{3/2}|_1^3 = \frac{\pi}{27} (730^{3/2} - 10^{3/2}). \)

18. We have a function of \( x \) rotating around the \( x \)-axis, so we use slices. Then \( V = \pi r^2 h, \) with \( h = dx \) and \( r = y = \frac{1}{(16 - x^2)^{3/4}}. \) Therefore, \( V = \int_0^2 \pi \left( \frac{1}{(16 - x^2)^{3/4}} \right)^2 dx = \int_0^2 \pi \frac{1}{(16 - x^2)^{3/2}} dx. \) Let \( x = 4 \sin \theta. \) Then \( dx = 4 \cos \theta \, d\theta. \) If \( x = 0, \theta = 0, \) and if \( x = 2, \sin \theta = \frac{1}{2} \) so \( \theta = \frac{\pi}{6}. \) Substituting in the integral yields

\[
\pi \int_0^{\pi/6} \frac{1}{(16 - 16 \sin^2 \theta)^{3/2}} \cdot 4 \cos \theta \, d\theta = \pi \int_0^{\pi/6} \frac{4 \cos \theta}{64 \cos^3 \theta} \, d\theta = \frac{\pi}{16} \int_0^{\pi/6} \sec^2 \theta \, d\theta = \frac{\pi}{16} \tan \theta|_0^{\pi/6} = \frac{\pi}{16} \left( \frac{\sqrt{3}}{3} - 0 \right) = \frac{\sqrt{3}\pi}{48}.
\]

19. The given fraction is improper, so we do long division to obtain \( \int \left( 1 + \frac{-4x + 4}{x^3 + 4x} \right) dx. \)

We use partial fractions on the remainder: \( -4x + 4 \) \( = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}. \) Clearing the fractions yields \(-4x + 4 = A(x^2 + 4) + (Bx + C)(x). \) If \( x = 0, 4 = 4A, \) or \( A = 1. \) Expanding with \( A = 1 \) yields \(-4x + 4 = x^2 + 4 + Bx^2 + Cx. \) From the coefficients of \( x^2, \) \( 0 = 1 + B, \) or \( B = -1. \) From the coefficients of \( x, -4 = C. \) Therefore, \( \int \frac{x^3 + 4}{x^3 + 4x} \, dx = \int \left( 1 + \frac{1}{x} - \frac{x}{x^2 + 4} - \frac{4}{x^2 + 4} \right) \, dx = x + \ln |x| - \frac{1}{2} \ln |x^2 + 4| - 4 \cdot \frac{1}{2} \arctan \left( \frac{x}{2} \right) + C. \)

(a) Can be done using the Integral Test, Comparison Test with \( \sum_{n=1}^{\infty} \frac{2}{n^3}, \) or Limit Comparison Test with \( \sum_{n=1}^{\infty} \frac{2}{n^3}. \)

(b) The Remainder Estimate for the Integral Test states that \( s - s_N \leq \int_N^{\infty} f(x) \, dx. \) Thus, \( s - s_9 \leq \int_9^{\infty} \frac{2}{(x+1)^3} \, dx. \)

\[
\int 2(x+1)^{-3} \, dx = -\frac{1}{2} (x+1)^{-2}, \quad \text{so} \quad s - s_9 \leq \lim_{a \to \infty} -\frac{1}{2(a+1)^2} + \frac{1}{2(10)^2}, \quad \text{or} \quad s - s_9 \leq \frac{1}{2(10)^2}
\]