MATH 152, FALL 2013
COMMON EXAM II - VERSION B

Print name (LAST, First): ___________________________ SECTION #: _________

INSTRUCTOR: ___________________________ SEAT #: _________

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PART I-MULTIPLE CHOICE
The use of any electronic device is prohibited. Mark the correct choice on your ScanTron using a No. 2 pencil. For your own records, also record your choices on your exam! Be sure to write your name, section and version letter of the exam on the ScanTron form. Each problem is worth 3 points.

1. Which statement is true about the sequence \( a_n = \ln(n^2 + 1) - \ln(3n^2 + 1) \)?
   (a) The sequence converges to \( \ln 3 \).
   (b) The sequence converges to \( \ln \left( \frac{1}{3} \right) \).
   (c) The sequence diverges.
   (d) The sequence converges to 0.
   (e) None of the other statements is true.

2. Which statement is true about the sequence \( a_n = \frac{(-1)^n(7n^2)}{n^3 + 1} \)?
   (a) The sequence converges to 7.
   (b) The sequence diverges.
   (c) None of the other statements is true.
   (d) The sequence converges to \( \frac{1}{7} \).
   (e) The sequence converges to 0.

3. Assuming the first term is \( a_1 \) and that the pattern of the first few terms continues, what is the general term \( a_n \) of the sequence \( \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \ldots \)?
   (a) \( a_n = \frac{1}{2} \)
   (b) None of these
   (c) \( a_n = \frac{n}{2n-1} \)
   (d) \( a_n = \frac{n+1}{2n+1} \)
   (e) \( a_n = \frac{2}{3} \left( \frac{9}{10} \right)^{n-1} \)
4. Find the sum of the series $4 + \frac{12}{5} + \frac{36}{25} + \frac{108}{125} + \frac{324}{625} + \cdots$
   
   (a) 9
   (b) 10
   (c) None of these
   (d) 8
   (e) The series diverges.

5. Which of the following gives the partial fraction decomposition of $\frac{x - 2}{(x - 4)^3(x^2 + 4x + 16)}$?
   
   (a) $\frac{A}{x - 4} + \frac{B}{(x - 4)^2} + \frac{C}{(x - 4)^3} + \frac{Dx + E}{x^2 + 4x + 16}$
   (b) $\frac{A}{(x - 4)^3} + \frac{B}{x^4 + 4x + 16}$
   (c) $\frac{A}{x - 4} + \frac{Bx + C}{(x - 4)^2} + \frac{Dx^2 + Ex + F}{(x - 4)^3} + \frac{Gx + H}{x^2 + 4x + 16}$
   (d) None of these
   (e) $\frac{Ax^4 + Bx^3 + Cx^2 + Dx + E}{x^5 - 8x^4 + 16x^3 - 64x^2 + 512x - 1024}$

6. Given a series $\sum_{n=1}^{\infty} a_n$ whose partial sums are given by $s_n = \frac{n + 8}{n + 2}$, which statement is true about the series?
   
   (a) The series converges to 1.
   (b) The series converges to 4.
   (c) The series converges to 3.
   (d) The series diverges.
   (e) None of the other statements is true.

7. Given the series in #6, which statement is true about the TERMS of the series $(a_n)$?
   
   (a) None of the other statements is true.
   (b) $a_n$ converges to 4.
   (c) $a_n$ converges to 0.
   (d) $a_n$ converges to 1.
   (e) $a_n$ diverges.
8. Evaluate \( \int_0^\infty 4xe^{-x^2} \, dx \).

(a) None of these  
(b) 2  
(c) –2  
(d) 0  
(e) The improper integral diverges

9. Which statement is true about the sequence defined by \( a_1 = 4, \ a_{n+1} = \frac{a_n}{a_n - 1} \)?

(a) The sequence converges to 0.  
(b) None of the other statements is true.  
(c) The sequence converges to 1.  
(d) The sequence diverges.  
(e) The sequence converges to 2.

10. Which statement is true about \( \int_0^3 \frac{1}{4x - 2} \, dx \)?

(a) The integral diverges.  
(b) The integral converges to \( \ln 10 - \ln 2 \).  
(c) The integral converges to \( \frac{1}{10} + \frac{1}{2} \).  
(d) None of the other statements is true.  
(e) The integral converges to \( \frac{1}{4} (\ln 10 - \ln 2) \).

11. Which of the following is an appropriate substitution to evaluate \( \int \frac{1}{\sqrt{x^2 - 8x + 32}} \, dx \)?

(a) \( x - 8 = \sqrt{32} \sec \theta \)  
(b) \( x - 8 = 32 \sec \theta \)  
(c) \( x - 4 = 16 \tan \theta \)  
(d) \( x - 4 = 4 \tan \theta \)  
(e) None of these

12. Find the sum of \( \sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)}) \).

(a) \( \frac{e^2}{e - 1} \)  
(b) The series diverges.  
(c) \( e \)  
(d) \( e - 1 \)  
(e) None of these
13. The surface area of a sphere of radius $r$ can be derived by rotating the semicircle $y = \sqrt{r^2 - x^2}$ about the $x$-axis. Which integral gives us the resulting area?

(a) $\int_0^r 2\pi x \sqrt{1 + \frac{(r-x)^2}{r^2 - x^2}} \, dx$

(b) $\int_0^r 2\pi \sqrt{r^2 - x^2} \sqrt{1 + \frac{(r-x)^2}{r^2 - x^2}} \, dx$

(c) None of these

(d) $\int_{-r}^r 2\pi x \sqrt{1 + \frac{x^2}{r^2 - x^2}} \, dx$

(e) $\int_{-r}^r 2\pi \sqrt{r^2 - x^2} \sqrt{1 + \frac{x^2}{r^2 - x^2}} \, dx$

14. Which of the series below is convergent?

(I) $\sum_{n=1}^{\infty} \frac{6n + 3}{n^3(n+1)}$

(II) $\sum_{n=1}^{\infty} \frac{6n + 3}{n(n+1)}$

(III) $\sum_{n=1}^{\infty} \frac{6n + 3}{n+1}$

(a) (I) only

(b) All 3 are convergent.

(c) All 3 are divergent.

(d) None of the other answers is correct.

(e) (I) and (II) only

15. Debris falls from an aircraft flying at 200 m/s at an altitude of 4500 meters. The trajectory of the debris can be modeled by $y = \frac{9}{2} - \frac{1}{8}x^2$, where $x$ and $y$ are in thousands of meters. Which integral gives the distance the debris travels from the point of release ($x = 0$) to the point it hits the ground?

(a) $\int_0^6 \sqrt{1 + \frac{x^2}{16}} \, dx$

(b) $\int_0^6 \sqrt{1 + \left(\frac{9}{2} - \frac{x}{4}\right)^2} \, dx$

(c) $\int_0^6 \left(\frac{9}{2} + \frac{x^2}{8}\right) \, dx$

(d) $\int_0^{4.5} \sqrt{1 + \left(\frac{9}{2} - \frac{x^2}{8}\right)^2} \, dx$

(e) $\int_0^{4.5} \left(\frac{9}{2} + \frac{x^2}{8}\right) \, dx$

PART II WORK OUT

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and BOX YOUR FINAL ANSWER. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

NOTE: The following formulas may or may not be useful on this exam:

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$
16. (9 points) Evaluate \( \int \frac{x^3 + 9}{x^3 + 9x} \, dx \).

17. (10 points) Find the volume of the solid formed by rotating the region bounded by \( y = \frac{1}{(4 - x^2)^{3/4}} \), \( y = 0 \), \( x = 0 \), and \( x = 1 \) about the \( x \)-axis.
18. (10 points) The curve $y = \sqrt{x}$ from the point $(1, 1)$ to the point $(27, 3)$ is rotated about the $y$-axis. Find the area of the resulting surface.

19. (10 points) Find the length of the curve parametrized by $x = 2t, \ y = e^t + e^{-t}, \ 0 \leq t \leq 3$. 
20. (8 points each)

(a) Is the series \( \sum_{n=1}^{\infty} \frac{4}{(n+1)^5} \) convergent or divergent? Clearly explain your reasoning.

(b) According to MATLAB (format long), the ninth partial sum, \( s_9 \approx 0.147629365378774 \). Use the Remainder Estimate for the Integral Test to estimate an upper bound on the error in using \( s_9 \) to approximate the sum of the series \( s \).
MATH 152 Exam II

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