1. **B** $a_n = \ln\left(\frac{n^2 + 1}{3n^2 + 1}\right)$. As $n \to \infty$, $\frac{n^2 + 1}{3n^2 + 1} \to \frac{1}{3}$, so $a_n \to \ln\left(\frac{1}{3}\right)$.

2. **E** $|a_n| = \frac{7n^2}{n^3 + 1}$. As $n \to \infty$, $|a_n| \to 0$, so $a_n \to 0$.

3. **D** Since the first term is $a_1$, the numerator of each term is $n + 1$ and the denominator (with slope = 2 and equal to 3 when $n = 1$) is $2n + 1$. Therefore, $a_n = \frac{n + 1}{2n + 1}$.

4. **B** The series is geometric with $a = 4$ and $r = \frac{3}{5}$. Since $|r| < 1$, the series converges to $\frac{a}{1 - r} = \frac{4}{1 - \frac{3}{5}} = 10$.

5. **A** The denominator contains a repeating linear factor and an irreducible quadratic factor, so the form of the partial fraction decomposition is $\frac{B}{x-4} + \frac{C}{(x-4)^2} + \frac{Dx+E}{x^2+4x+16}$.

6. **A** Since $s_n \to 1$, the series converges to 1.

7. **C** By \#6 the series is convergent. The Test for Divergence therefore says that $a_n \to 0$.

8. **B** Let $u = -x^2$. Then $du = -2x\,dx$ so $\int_0^\infty 4xe^{-x^2}\,dx = \lim_{a \to \infty} -2e^{-x^2}|_0^a = \lim_{a \to \infty} -2e^{-a^2} + 2 = 2$.

9. **D** $a_1 = 4$, $a_2 = \frac{4}{4 - 1} = 4$, $a_3 = \frac{4/3}{4/3 - 1} = 4$, $a_4 = \frac{4}{3}$, so the sequence oscillates between 4 and $\frac{4}{3}$ and therefore diverges.

10. **A** $\int_0^3 \frac{1}{4x-2}\,dx = \int_0^{1/2} \frac{1}{4x-2}\,dx + \int_{1/2}^3 \frac{1}{4x-2}\,dx$

\begin{align*}
&= \lim_{a \to 1/2^-} \int_0^a \frac{1}{4x-2}\,dx + \lim_{a \to 1/2^+} \int_a^3 \frac{1}{4x-2}\,dx=
\lim_{a \to 1/2^-} \frac{1}{4}(\ln|4a-2| - \ln|2|) + \lim_{a \to 1/2^+} \frac{1}{4}(\ln|4a-2| - \ln|4a-2|)
\end{align*}

which diverges since $\lim_{a \to 1/2} \ln|4a-2| = -\infty$.

11. **D** Complete the square to yield $\int \frac{1}{\sqrt{(x-4)^2 + 16}}$.

Then the desired substitution is $x = 4 \tan \theta$.

12. **D** $s_n = \sum_{i=1}^n (e^{1/i} - e^{1/(i+1)}) = (e^1 - e^{1/2}) + (e^{1/2} - e^{1/3}) + \cdots + (e^{1/(n-1)} - e^{1/n}) + (e^{1/n} - e^{1/(n+1)})$.

Cancelling terms (telescoping) yields $s_n = e - e^{1/(n+1)}$. As $n \to \infty$, $\frac{1}{n+1} \to 0$, so the series converges to $e - e^0 = e - 1$.

13. **E** $S = \int 2\pi R ds$. Since the curve is rotated about the $x$-axis, $R = y$. Choosing to evaluate arclength with respect to $x$ (meaning $R = \sqrt{r^2 - x^2}$), $ds = \sqrt{1 + \left(\frac{1}{2}(r^2 - x^2)^{-1/2}(-2x)\right)^2}\,dx = \sqrt{1 + \frac{x^2}{r^2 - x^2}}\,dx$. Therefore, $S = \int_{-r}^r 2\pi \sqrt{r^2 - x^2}\sqrt{1 + \frac{x^2}{r^2 - x^2}}\,dx$.

14. **A** $\sum_{n=1}^\infty \frac{6n + 3}{n^2(n+1)}$ is convergent by the Comparison (or Limit Comparison) Test with $\sum_{n=1}^\infty \frac{6}{n^2}$. $\sum_{n=1}^\infty \frac{6n + 3}{n(n+1)}$ is divergent by the Limit Comparison Test with $\sum_{n=1}^\infty \frac{6}{n}$, $\sum_{n=1}^\infty \frac{6n + 3}{n + 1}$ is divergent by the Test for Divergence.

15. **A** The debris hits the ground when $y = 0$, or $0 = \frac{9}{2} - \frac{1}{8}x^3$, $x^3 = 36$, or $x = 6$.

\begin{align*}
\frac{dy}{dx} &= -\frac{1}{4}x, \text{ so the length of the curve is } s &= \int_0^6 \sqrt{1 + \left(-\frac{1}{4}x\right)^2}\,dx = \int_0^6 \sqrt{1 + \frac{x^2}{16}}\,dx.
\end{align*}
16. The given fraction is improper, so we do long division to obtain \( \int \left( 1 + \frac{-9x + 9}{x^3 + 9x} \right) dx \).

We use partial fractions on the remainder:

\[
\frac{-9x + 9}{x(x^2 + 9)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 9}
\]

Clearing the fractions yields

\[-9x + 9 = A(x^2 + 9) + (Bx + C)x.\]

If \( x = 0, \) \( 9 = 9A, \) or \( A = 1. \) Expanding with \( A = 1 \) yields

\[-9x + 9 = x^2 + 9 + Bx + C.\]

From the coefficients of \( x^2, \) \( 0 = 1 + B, \) or \( B = -1. \)

From the coefficients of \( x, -9 = C. \) Therefore,

\[
\int \frac{x^3 + 9}{x^3 + 9x} dx = \int \left( 1 + \frac{-9x + 9}{x^3 + 9} \right) dx = \int \left( 1 + \frac{1}{x} - \frac{x}{x^2 + 9} \right) dx = x + \ln |x| - \frac{1}{2} \ln |x^2 + 9| - \frac{9}{3} \arctan \left( \frac{x}{3} \right) + C.
\]

17. We have a function of \( x \) rotating around the \( x \)-axis, so we use slices. Then \( V = \pi r^2 h, \) with \( h = dx \) and \( r = y = \left( 4 - x^2 \right)^{3/2}. \) Therefore,

\[
V = \int_{0}^{1} \pi \left( \frac{1}{4 - x^2} \right)^{3/2} dx = \int_{0}^{1} \pi \frac{1}{4 - x^2} dx.
\]

Let \( x = 2 \sin \theta. \) Then \( dx = 2 \cos \theta d\theta. \) If \( x = 0, \) \( \theta = 0, \) and if \( x = 1, \) \( \sin \theta = \frac{1}{2} \) so \( \theta = \pi / 6. \) Substituting in the integral yields

\[
\pi \int_{0}^{\pi/6} \frac{1}{4 - 4(1 - \cos^2 \theta)^{3/2}} 2 \cos \theta d\theta = \pi \int_{0}^{\pi/6} \frac{2 \cos \theta}{8 \cos^3 \theta} d\theta = \pi \int_{0}^{\pi/6} \frac{\tan \theta}{\sec^2 \theta} d\theta = \frac{\pi}{4} \tan \frac{\pi}{6} = \frac{\pi}{4} \left( \frac{\sqrt{3}}{3} - 0 \right) = \sqrt{3} \pi / 12.
\]

18. \( S = \int 2\pi r ds. \) Since the curve is rotating about the \( y \)-axis, \( r = x. \) It is easier to integrate with respect to \( y, \) so \( r = y^3 \) and \( ds = \sqrt{\left( \frac{dx}{dy} \right)^2 + 1} dy = \sqrt{(3y^2)^2 + 1} dy = \sqrt{9y^4 + 1} dy. \) Therefore,

\[
S = \int_{1}^{3} 2\pi y^3 \sqrt{9y^4 + 1} dy.
\]

Let \( u = 9y^4 + 1. \) Then

\[
du = 36y^3 dy, \quad S = 2\pi \left( \frac{1}{36} \right) \left( \frac{2}{3} \right) (9y^4 + 1)^{3/2} \bigg|_{1}^{3} = \frac{\pi}{27} (730^{3/2} - 10^{3/2}).
\]

19. \( ds = \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2}. \quad \frac{dx}{dt} = 2, \)

and \( \frac{dy}{dt} = e^t - e^{-t}. \) Then

\[
s = \int_{0}^{3} \sqrt{2^2 + (e^t - e^{-t})^2} dt = \int_{0}^{3} \sqrt{4 + e^{2t} - 2 + e^{-2t}} dt = \int_{0}^{3} \sqrt{e^{2t} + 2 + e^{-2t}} dt = \int_{0}^{3} (e^t + e^{-t}) dt = e^t - e^{-t} \bigg|_{0}^{3} = (e^3 - e^{-3}) - (1 - 1).
\]

(a) Can be done using the Integral Test, Comparison Test with \( \sum_{n=1}^{\infty} \frac{4}{n^n}, \) or Limit Comparison Test with \( \sum_{n=1}^{\infty} \frac{4}{n^5}. \)

(b) The Remainder Estimate for the Integral Test states that \( s - s_N \leq \int_{N}^{\infty} f(x) dx. \)

Thus,

\[ s - s_N \leq \int_{N}^{\infty} \frac{4}{(x + 1)^5} dx. \]

\[ \int 4(x + 1)^{-5} dx = -(x + 1)^{-4}, \quad \text{so} \]

\[ s - s_N \leq \lim_{a \to \infty} -\frac{1}{(a + 1)^4} + \frac{1}{104}, \quad \text{or} \]

\[ s - s_N \leq \frac{1}{104}. \]