LAST NAME(print): _______________ FIRST NAME(print): _______________

INSTRUCTOR: ____________________

SECTION NUMBER: ______________

ROW NUMBER: ________________

DIRECTIONS:

1. The use of a cell phone, calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ________________________

Some integrals that may or may not be useful.

\[
\int \sec x \, dx = \ln | \sec x + \tan x | + C \\
\int \csc x \, dx = \ln | \csc x - \cot x | + C \\
\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln | \sec x + \tan x | + C \\
\int \csc^3 x \, dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln | \csc x - \cot x | + C
\]
PART I: Multiple Choice. 4 points each

1. Given that the sequence $a_n$ converges to 2 and the sequence $b_n$ converges to -3, what conclusion can be made about the sequence $\frac{a_n b_n}{2 + 3b_n}$?

   (a) None of these.
   (b) Converges to 6/29
   (c) Converges to 3/7
   (d) Converges to -6/29
   (e) Converges to -3/7

2. Compute $\int 4 \sin^2 \theta d\theta$

   (a) $2\theta + \sin(2\theta) + C$
   (b) $4\theta + 2 \sin(2\theta) + C$
   (c) None of these.
   (d) $2\theta - \sin(2\theta) + C$
   (e) $\frac{4}{3} \sin^3 \theta + C$

3. Compute $\int_1^3 \frac{3x^2 + 2x + 3}{x^2 + 1} \, dx$

   (a) $6 + 2 \arctan(3) - 2 \arctan(1)$
   (b) $6 + \ln(10) - \ln(2)$
   (c) $6 - \ln(10) + \ln(2)$
   (d) $6 + \ln(10 + \ln(2))$
   (e) None of these.
4. Which of the following is the form of the partial-fraction decomposition for the rational function \[ \frac{3x + 7}{(x - 1)(x^2 + 2x - 3)(x^2 + 4)} \]

(a) \[ \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 2x - 3} + \frac{Dx + E}{x^2 + 4} \]
(b) None of these.
(c) \[ \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 3} + \frac{Dx + E}{x^2 + 4} \]
(d) \[ \frac{A}{x - 1} + \frac{Bx + C}{(x - 1)^2} + \frac{D}{x + 3} + \frac{Ex + F}{x^2 + 4} \]
(e) \[ \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 3} + \frac{D}{x + 2} + \frac{D}{x - 2} \]

5. The integral \( \int_{-1}^{2} \frac{3}{x^4} \, dx \)

(a) converges to \( -\frac{9}{8} \)
(b) converges to \( -\frac{48}{16} \)
(c) None of these.
(d) converges to \( \frac{2}{8} \)
(e) diverges

6. Decide on the convergence/divergence of each of the following improper integrals.

(I) \( \int_{2}^{\infty} \frac{x + 2}{x^4} \, dx \)  (II) \( \int_{1}^{\infty} \frac{1}{\sqrt[3]{x^3}} \, dx \)

(a) (I) is convergent and (II) is convergent
(b) (I) is convergent and (II) is divergent
(c) (I) is divergent and (II) is divergent
(d) (I) is divergent and (II) is convergent
(e) More information is needed to make a decision.
7. The curve $C$ is given by the equation $y = 4 + 3x^2$. Which integral gives the length of the curve from the point $(0, 4)$ to the point $(2, 16)$?

(a) $\int_{0}^{2} \sqrt{1 + 36x^2} \, dx$

(b) $\int_{2}^{4} \sqrt{1 + 36x^2} \, dx$

(c) $\int_{0}^{16} \sqrt{1 + 6x^2} \, dx$

(d) $\int_{4}^{16} \sqrt{1 + 6x^2} \, dx$

(e) None of these.

8. Assume that the sequence $\{a_n\}$ is bounded, increasing and given by

$$a_1 = 3 \quad \text{and} \quad a_{n+1} = 6 - \frac{8}{a_n}$$

for all positive integers $n$. Determine if the sequence is convergent or divergent.

(a) Divergent

(b) Convergent to 2

(c) Convergent to 4

(d) Convergent to 6

(e) Convergent to 8

9. After an appropriate substitution, the integral $\int \sqrt{9 - x^2} \, dx$ is equivalent to which of the following?

(a) $3 \int \cos \theta \, d\theta$

(b) $9 \int \sec \theta \tan^2 \theta \, d\theta$

(c) $9 \int \sec^3 \theta \, d\theta$

(d) $9 \int \cos^2 \theta \, d\theta$

(e) $3 \int \tan \theta \, d\theta$
10. After an appropriate substitution, the integral \( \int \frac{\sqrt{9x^2 + 4}}{x} \, dx \) is equivalent to which of the following?

(a) \( \int \frac{\sec^3 \theta}{\tan \theta} \, d\theta \)
(b) \( \int \frac{3 \sec \theta}{\tan \theta} \, d\theta \)
(c) \( \int \frac{2 \sec \theta}{\tan \theta} \, d\theta \)
(d) None of these.
(e) \( \int \frac{2 \sec^3 \theta}{\tan \theta} \, d\theta \)

11. The curve C is given by the parametric equations \( x = t^2, \ y = t^2 + t \) for \( 0 \leq t \leq 1 \). Which integral gives the surface area obtained when rotating C about the y-axis?

(a) \( 2\pi \int_0^1 \sqrt{1 + 4t + 8t^2} \, dt \)
(b) None of these.
(c) \( 2\pi \int_0^1 t^2 \sqrt{1 + 4t + 8t^2} \, dt \)
(d) \( 2\pi \int_0^1 (t^2 + t) \sqrt{1 + 4t + 8t^2} \, dt \)
(e) \( 2\pi \int_0^1 t^2 \sqrt{t^2 + 2t^3 + 2t^4} \, dt \)

12. Compute \( \int \sin^3(2x) \, dx \)

(a) None of these.
(b) \( \frac{1}{2} \cos(2x) - \frac{1}{6} \cos^3(2x) + C \)
(c) \( -\cos(2x) + \frac{1}{3} \cos^3(2x) + C \)
(d) \( \cos(2x) - \frac{1}{3} \cos^3(2x) + C \)
(e) \( -\frac{1}{2} \cos(2x) + \frac{1}{6} \cos^3(2x) + C \)
13. Which sequence is both bounded and increasing?

- (a) \( a_n = 1 - \frac{2}{n} \)
- (b) \( a_n = \ln n \)
- (c) \( a_n = \sin(2n\pi) \)
- (d) \( a_n = e^{-n} \)
- (e) None of these.

14. The integral \( \int_3^\infty \frac{3 + \sin x}{x} \, dx \) is

- (a) convergent by comparison with \( \int_3^\infty \frac{2}{x} \, dx \)
- (b) divergent by comparison with \( \int_3^\infty \frac{3}{x} \, dx \)
- (c) divergent by comparison with \( \int_3^\infty \frac{4}{x} \, dx \)
- (d) divergent by comparison with \( \int_3^\infty \frac{2}{x} \, dx \)
- (e) convergent by comparison with \( \int_3^\infty \frac{4}{x} \, dx \)

15. The sequence \( a_n = \frac{(-1)^n n^2}{2n^2 + 5} \)

- (a) Converges to \( \frac{1}{2} \)
- (b) Diverges
- (c) Converges to \( \frac{-1}{2} \)
- (d) None of these.
- (e) Converges to 0
PART II WORK OUT

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and Box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (8 points) Compute \[ \int_2^\infty \left( \frac{7}{7x+2} - \frac{4}{4x+1} \right) \, dx = \lim_{b \to \infty} \int_2^b \frac{7}{7x+2} - \frac{4}{4x+1} \, dx = \lim_{b \to \infty} \left[ \ln(7x+2) - \ln(4x+1) \right]_2^b = \lim_{b \to \infty} \ln(7b+2) - \ln(4b+1) - \left( \ln(16) - \ln(9) \right) = \lim_{b \to \infty} \ln\left( \frac{7b+2}{4b+1} \right) - \ln\left( \frac{16}{9} \right) = \ln\left( \frac{7}{4} \right) - \ln\left( \frac{16}{9} \right) \]

Since \( \frac{7b+2}{4b+1} \to \frac{7}{4} \) as \( b \to \infty \)

17. (8 points) Compute \[ \int \frac{\sec^4 x}{\tan x} \, dx = \int \frac{\sec^2 x \cdot \sec^2 x}{\tan x} \, dx = \int \frac{(\tan^2 x + 1) \cdot \sec^2 x}{\tan x} \, dx \]

\( u = \tan x \)
\( du = \sec^2 x \, dx \)

\[ = \int \frac{u^2 + 1}{u} \, du = \int u + \frac{1}{u} \, du = \frac{u^2}{2} + \ln |u| + C = \frac{\tan^2 x}{2} + \ln |\tan x| + C \]
18. (8 points) Compute \( \int \frac{1}{\sqrt{x^2 + 4x}} \, dx \)

\[
= \int \frac{1}{\sqrt{(x+2)^2 - 1}} \, dx = \int \frac{2 \sec \theta \tan \theta}{\sqrt{4 \sec^2 \theta - 4}} \, d\theta = \int \frac{2 \sec \theta \tan \theta}{2 \tan \theta} \, d\theta = \int \sec \theta \, d\theta
\]

\[
x + 2 = 2 \sec \theta
\]

\[
dx = 2 \sec \theta \tan \theta \, d\theta
\]

\[
\frac{x+2}{2} + \frac{\sqrt{(x+2)^2 - 1}}{2} + C
\]

19. The curve \( C \) is given by the equation \( x = \frac{1}{3}(y^2 + 2)^{3/2} \) for \( 0 \leq y \leq 1 \).

(a) (5 points) Set up the integral that will compute the surface area obtained when rotating \( C \) about the \( x \)-axis.

\[
SA = \int 2\pi y \sqrt{1 + (x')^2} \, dy = \int 2\pi y \sqrt{1 + \left( \frac{3}{2}(y^2 + 2)^{3/2} \right)^2} \, dy
\]

(b) (2 points) Compute the integral in part (a).

\[
SA = \int 2\pi y \sqrt{1 + y^2(y^2 + 2)} \, dy = \int 2\pi y \sqrt{1 + y + 2y^2} \, dy
\]

\[
= \int 2\pi y \sqrt{y^2 + 2y^2 + 1} \, dy = \int 2\pi y \sqrt{(y^2 + 1)^2} \, dy
\]

\[
= \int 2\pi y \, (y^2 + 1) \, dy = \int 2\pi (y^3 + y) \, dy = 2\pi \left( \frac{y^4}{4} + \frac{y^2}{2} \right) \bigg|_0^1
\]

\[
= 2\pi \left( \frac{1}{4} + \frac{1}{2} \right) = 2\pi \left( \frac{3}{4} \right) = \frac{6\pi}{4} = \frac{3\pi}{2}
\]
20. (9 points) Compute \( \int \frac{2x^3 + 11x^2 + 18}{x^2(x^2 + 9)} \, dx \)

\[
\frac{2x^3 + 11x^2 + 18}{x^2(x^2 + 9)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C x + D}{x^2 + 9}
\]

\[
2x^3 + 11x^2 + 18 = A x(x^2 + 9) + B (x^2 + 9) + (C x + D) x^2
\]

\[
2x^3 + 11x^2 + 18 = A (x^3 + 9x) + B (x^2 + 9) + C x^3 + D x^2
\]

\[
\begin{align*}
2x^3 & = A + C = 2 \\
x^2 & = B + D = 11 \\
x & = 9A = 0 \implies A = 0 \implies C = 2 \\
\text{const} & = 9B = 18 \implies B = 2 \implies D = 9
\end{align*}
\]

\[
\int \frac{2}{x^2} + \frac{2x + 9}{x^2 + 9} \, dx = \int \frac{2}{x^2} + \frac{2x}{x^2 + 9} + \frac{9}{x^2 + 9} \, dx
\]

\[
= -\frac{2}{x} + \ln(x^2 + 9) + 3 \arctan\left(\frac{x}{3}\right) + C
\]

\[
\int \frac{4}{x^2 + 9} \, dx = \int \frac{4}{9(x^2 + 1)} \, dx = \frac{1}{3} \int \frac{1}{u^2 + 1} \, du
\]

\[
U = \frac{x}{3} \\
du = \frac{1}{3} \, dx \\
3 \, du = dx
\]

\[
= 3 \arctan(u) = 3 \arctan\left(\frac{x}{3}\right)
\]