DIRECTIONS:

1. The use of a cell phone, calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________
PART I: Multiple Choice. 4 points each

1. Write \( f(x) = \frac{1}{1 + 9x^2} \) as a power series centered at 0. Include the radius of convergence.

   (a) \( \sum_{n=0}^{\infty} 9^n x^{2n}, R = \frac{1}{3} \)
   (b) \( \sum_{n=0}^{\infty} 9^n x^{2n}, R = \frac{1}{9} \)
   (c) \( \sum_{n=0}^{\infty} (-9)^n x^{2n}, R = \frac{1}{3} \)
   (d) \( \sum_{n=0}^{\infty} (-9)^n x^{2n}, R = \frac{1}{9} \)
   (e) None of these.

2. For which series is the ratio test inconclusive?

   (a) None of these.
   (b) \( \sum_{n=1}^{\infty} ne^{-n} \)
   (c) \( \sum_{n=1}^{\infty} \frac{n+2}{n!} \)
   (d) \( \sum_{n=1}^{\infty} \frac{n^2}{3^n} \)
   (e) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^5} \)

3. Let \( \sum_{n=1}^{\infty} a_n \) be a series whose \( n \)th partial sum is \( s_n = \frac{3n^2 + 5}{5n^2 + 2} \). The series

   (a) converges to \( \frac{8}{7} \)
   (b) converges to \( \frac{3}{5} \)
   (c) converges to 2.5
   (d) diverges
   (e) None of these.

4. Compute the sum of the series \( \sum_{n=1}^{\infty} \frac{4^{n+1}}{5^n} \).

   (a) This series diverges.
   (b) 20
   (c) None of these.
   (d) 12
   (e) 16
5. Which of the following is the MacLaurin series for \( f(x) = x^3 \cos(x) \)?

(a) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+1)!} \)

(b) None of these.

(c) \( \sum_{n=0}^{\infty} \frac{x^{n+3}}{n!} \)

(d) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n)!} \)

(e) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{2n+1} \)

6. The series \( \sum_{i=1}^{\infty} \left( e^{1/i} - e^{1/(i+1)} \right) \)

(a) converges to \( e \)

(b) None of these.

(c) converges to \( e - 1 \)

(d) diverges

(e) converges to 0

7. Find the radius of convergence of the series \( \sum_{n=1}^{\infty} \frac{n!(2x-3)^n}{5^n} \).

(a) 0

(b) \( \frac{3}{2} \)

(c) \( \frac{5}{2} \)

(d) \( \infty \)

(e) None of these.

8. Consider the 5th partial sum of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \) as an approximation. Use the alternating series rule to obtain an upper bound on the absolute value of the error.

(a) \( \frac{-1}{5 \times 7^5} \)

(b) None of these.

(c) \( \frac{1}{5 \times 7^5} \)

(d) \( \frac{1}{4 \times 7^4} \)

(e) \( \frac{1}{6 \times 7^6} \)
9. Find the 23th derivative at \( x = 0 \) for \( f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(n+2)} x^n \)

(a) None of these.
(b) \( f^{(23)}(0) = \frac{23!}{3^{23}(25)} \)
(c) \( f^{(23)}(0) = \frac{-1}{3^{23}(25)} \)
(d) \( f^{(23)}(0) = \frac{-23!}{3^{23}(25)} \)
(e) \( f^{(23)}(0) = \frac{23}{3^{23}(25)} \)

10. Which series converges absolutely?

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \)
(b) \( \sum_{n=1}^{\infty} \frac{(-2)^n}{3^{n+1}} \)
(c) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \)
(d) None of these.
(e) \( \sum_{n=1}^{\infty} \frac{(-1)^n e^n}{e^{n+1}} \)

11. Which series converges but does not converge absolutely?

(a) None of these.
(b) \( \sum_{n=1}^{\infty} \frac{1}{n^2} \)
(c) \( \sum_{n=0}^{\infty} \left( -\frac{4}{3} \right)^n \)
(d) \( \sum_{n=1}^{\infty} (-1)^n \left( 1 - \frac{1}{n^2} \right) \)
(e) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \)
12. The series \( f(x) \) has a radius of convergence of 4. The series \( g(x) \) has a radius of convergence of 2. For which of these intervals will the series represented by \( h(x) = f(x) + g(x) \) converge, i.e. be defined?

\[
f(x) = \sum_{n=0}^{\infty} c_n (x - 2)^n \quad g(x) = \sum_{n=0}^{\infty} d_n (x - 5)^n \quad h(x) = \sum_{n=0}^{\infty} \left( c_n (x - 2)^n + d_n (x - 5)^n \right)
\]

(a) \( 3 < x < 6 \)
(b) \( -2 < x < 7 \)
(c) \( 3 < x < 7 \)
(d) \( 2 < x < 5 \)
(e) \( -2 \leq x \leq 7 \)

13. Find the radius of convergence of the series \( \sum_{n=1}^{\infty} \frac{x^{2n}}{3^n} \).

(a) 3
(b) 0
(c) \( \sqrt{3} \)
(d) \( \infty \)
(e) None of these.

14. Find the sum of the series \( \sum_{n=0}^{\infty} \frac{(-2)^n}{n!} \).

(a) \( e^2 \)
(b) None of these
(c) \( \sin(-2) \)
(d) \( e^{-2} \)
(e) 0

15. Find the center and radius of the sphere: \( x^2 - 6x + y^2 + (z + 4)^2 = 100 \)

(a) center: \( (3,0,-4) \), radius = \( \sqrt{109} \)
(b) center: \( (-3,0,4) \), radius = 109
(c) None of these.
(d) center: \( (3,0,-4) \), radius = 109
(e) center: \( (-3,0,4) \), radius = \( \sqrt{109} \)
PART II WORK OUT

**Directions**: Present your solutions in the space provided. *Show all your work* neatly and concisely and *Box your final answer*. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (4 points each) Determine whether the series converges and give reasons for your answer.

   (a) \[ \sum_{n=1}^{\infty} \frac{3^n}{1 + 2^{2n}} \]

   (b) \[ \sum_{n=1}^{\infty} \cos \left( \frac{1}{n} \right) \]

17. (5 points) Compute the limit using an appropriate series.

   \[ \lim_{x \to 0} \frac{e^{-2x^2} - 1 + 2x^2}{5x^4} = \]
18. (6 points each) Find the Maclaurin series for each function and its radius of convergence.

(a) \( f(x) = \ln(4 + x) \)

(b) \( f(x) = \frac{x}{(1 - 3x)^2} \)
19. Consider the function \( f(x) = \ln(x) \).

(a) (4 points) Find the 3rd degree Taylor polynomial \( T_3(x) \) for \( f(x) \) about \( x = 4 \).

(b) (4 points) If this polynomial \( T_3 \) is used to approximate \( f(x) \) on the interval \( 1 \leq x \leq 7 \), estimate the maximum error, \( |R_3| \), in this approximation using Taylor’s Inequality.

\[
|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1} \quad \text{where} \quad |f^{(n+1)}(x)| \leq M \quad \text{For} \quad 1 \leq x \leq 7.
\]
20. (7 points) Find the radius of convergence and the interval of convergence of the power series.

\[ \sum_{n=1}^{\infty} \frac{(-1)^n (2x + 1)^n}{n5^n} \]