DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-20), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 3 points.

4. In Part 2 (Problems 21-24), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: __________________________

DO NOT WRITE BELOW!

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1. The integral $\int_{0}^{\infty} e^{-2x} \, dx$

   (a) converges to $\frac{1}{2}$
   (b) converges to 0
   (c) converges to $\frac{1}{4}$
   (d) converges to 2
   (e) diverges

2. Find the surface area obtained by rotating the curve $x = \sin(2t), \ y = \cos(2t), \ 0 \leq t \leq \frac{\pi}{4},$ about the $x$-axis.

   (a) $8\pi$
   (b) $4\pi$
   (c) $\sqrt{2}\pi$
   (d) $2\sqrt{2}\pi$
   (e) $2\pi$

3. Find the limit $L$ of the sequence $a_n = \ln(n) - \ln(3n + 2)$.

   (a) $L = 0$
   (b) $L = \ln \left( \frac{1}{3} \right)$
   (c) $L = \ln \left( \frac{1}{2} \right)$
   (d) $L = \ln \left( \frac{1}{5} \right)$
   (e) The sequence diverges to $-\infty$. 
4. Find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{6^n}$.

(a) 1/5  
(b) 5/6  
(c) 2/3  
(d) 6/5  
(e) 1

5. A sequence is defined recursively by $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = \sqrt{3}a_n$. Given that this sequence converges to a positive number, find the limit of the sequence.

(a) \(\sqrt{3}\)  
(b) 1/3  
(c) 3  
(d) 6  
(e) 2/3

6. Which of the following integrals gives the surface area obtained by rotating the curve $y = \ln(4x)$, $1 \leq x \leq 3$, about the y-axis?

(a) $\int_1^3 2\pi x \sqrt{1 + \frac{1}{x^2}} \, dx$  
(b) $\int_1^3 2\pi \ln(4x) \sqrt{1 + \frac{1}{x^2}} \, dx$  
(c) $\int_1^3 2\pi x \sqrt{1 + \frac{1}{16x^2}} \, dx$  
(d) $\int_1^3 2\pi \ln(4x) \sqrt{1 + \frac{1}{16x^2}} \, dx$  
(e) $\int_1^3 2\pi x \sqrt{1 + \ln^2 4x} \, dx$
7. Find the length of the curve \( y = \frac{2}{3}(x - 1)^{3/2} \), \( 1 \leq x \leq 4 \).

(a) \( 2\sqrt{3} \)
(b) \( 9 \)
(c) \( \frac{12}{5\sqrt{3}} \)
(d) \( \frac{14}{3} \)
(e) \( \sqrt{3} \)

8. Consider the series \( \sum_{n=1}^{\infty} a_n \) whose \( n \)-th partial sum is given by \( s_n = \frac{2}{3 - e^{-2n}} \). What is \( \sum_{n=1}^{\infty} a_n \)?

(a) \( 0 \)
(b) \( 2 \)
(c) \( 1 \)
(d) \( \frac{1}{3} \)
(e) \( \frac{2}{3} \)

9. The complete partial fraction decomposition of \( f(x) = \frac{x^2 + 4}{(x + 2)^2(x^2 - 1)(4x^2 + 5x + 3)} \) is

(a) \( \frac{A}{(x + 2)^2} + \frac{B}{x^2 - 1} + \frac{Cx + D}{4x^2 + 5x + 3} \)
(b) \( \frac{A}{x + 2} + \frac{B}{(x + 2)^2} + \frac{Cx^2 + D}{x^2 - 1} + \frac{Ex + F}{4x^2 + 5x + 3} \)
(c) \( \frac{A}{x + 2} + \frac{B}{(x + 2)^2} + \frac{C}{x - 1} + \frac{D}{x + 1} + \frac{Ex + F}{4x^2 + 5x + 3} \)
(d) \( \frac{A}{(x + 2)^2} + \frac{B}{x - 1} + \frac{C}{x + 1} + \frac{Dx + E}{4x^2 + 5x + 3} \)
(e) None of the above
10. The sequence $a_n = (-1)^n \left(1 + \frac{1}{n^2}\right)$

(a) converges to 1  
(b) converges to 0  
(c) converges to 2  
(d) diverges  
(e) converges to $-1$

11. Which of the following integrals is equivalent to $\int_{-1}^{2} \sqrt{x^2 + 2x + 10} \, dx$?

(a) $9 \int_{0}^{\pi/4} (1 + \tan \theta) \sec^2 \theta \, d\theta$  
(b) $9 \int_{0}^{\pi/4} \sec^3 \theta \, d\theta$  
(c) $9 \int_{0}^{\pi/4} \sec \theta \, d\theta$  
(d) $9 \int_{0}^{\pi/2} \sec^3 \theta \, d\theta$  
(e) $\int_{-1}^{2} (x + 4) \, dx$

12. Which of the following integrals gives the length of the curve $y = \ln(\cos(x))$, $0 \leq x \leq \frac{\pi}{4}$?

(a) $\int_{0}^{\pi/4} \sec x \, dx$  
(b) $\int_{0}^{\pi/4} \sqrt{1 + \csc^2 x} \, dx$  
(c) $\int_{0}^{\pi/4} (1 + \tan x) \, dx$  
(d) $\int_{0}^{\pi/4} \sqrt{1 + \sec^2 x} \, dx$  
(e) None of these
13. Which statement is true about the integral $\int_{0}^{4} \frac{2}{(x-3)^2} \, dx$?

(a) diverges  
(b) converges to $\frac{8}{3}$  
(c) converges to $\frac{4}{3}$  
(d) converges to $-\frac{8}{3}$  
(e) converges to $-\frac{4}{3}$

14. $\int \frac{dx}{x(x-3)^2} =$

(a) $-\frac{1}{9(x-3)} - \frac{1}{3} \ln |x-3| + \frac{1}{9} \ln |x| + C$
(b) $-\frac{1}{3(x-3)} - \frac{1}{9} \ln |x-3| + \frac{1}{9} \ln |x| + C$
(c) $\frac{1}{3(x-3)} - \frac{1}{9} \ln |x-3| + \frac{1}{9} \ln |x| + C$
(d) $\frac{1}{3(x-3)} + \frac{1}{9} \ln |x-3| - \frac{1}{9} \ln |x| + C$
(e) $-\frac{1}{3} \ln (x-3)^2 - \frac{1}{9} \ln |x-3| + \frac{1}{9} \ln |x| + C$

15. Find the sum $S$ of the geometric series $S = \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \cdots$

(a) $S = 3$  
(b) $S = 2$  
(c) $S = \frac{2}{3}$  
(d) $S = \frac{4}{15}$  
(e) $S = \frac{4}{3}$
16. The improper integral \( \int_1^e \frac{dx}{x \ln x} \)

(a) diverges to \( \infty \)
(b) diverges to \( -\infty \)
(c) converges to 1
(d) converges to \(-1\)
(e) converges to \(\frac{1}{e} - 1\)

17. \( \int_1^2 \frac{x^2 + 1}{x^2 + x} \, dx = \)

(a) \(1 - 3 \ln 2 + 2 \ln 3\)
(b) \(2 - 3 \ln 2 + 2 \ln 3\)
(c) \(2 + 3 \ln 2 - 2 \ln 3\)
(d) \(1 + 3 \ln 2 - 2 \ln 3\)
(e) None of the above.

18. Which of the following statements is true regarding integral \( \int_1^\infty \frac{dx}{x + e^{5x}} \)?

(a) Converges since \( \int_1^\infty \frac{dx}{x + e^{5x}} < \int_1^\infty \frac{dx}{x} \), which converges.
(b) Diverges since \( \int_1^\infty \frac{dx}{x + e^{5x}} < \int_1^\infty \frac{dx}{x} \), which diverges.
(c) Diverges since \( \int_1^\infty \frac{dx}{x + e^{5x}} > \int_1^\infty \frac{dx}{x} \), which diverges.
(d) Converges to 0.
(e) Converges since \( \int_1^\infty \frac{dx}{x + e^{5x}} < \int_1^\infty \frac{dx}{e^{5x}} \), which converges.
19. Which of the following integrals is equivalent to \[ \int \sqrt{4x^2 - 9} \, dx \]?

(a) \[ 2 \int \sec \theta \tan^2 \theta \, d\theta \]
(b) \[ \frac{9}{2} \int \sec \theta \tan^2 \theta \, d\theta \]
(c) \[ \frac{9}{2} \int \tan \theta \, d\theta \]
(d) \[ \frac{9}{2} \int \sec^2 \theta \tan \theta \, d\theta \]
(e) \[ 2 \int \sec^2 \theta \tan \theta \, d\theta \]

20. Which of the following series diverges by the Test for Divergence?

(a) \[ \sum_{n=1}^{\infty} \frac{\ln n}{n} \]
(b) \[ \sum_{n=1}^{\infty} \sin \left( \frac{\pi}{2} - \frac{1}{n} \right) \]
(c) \[ \sum_{n=1}^{\infty} \frac{n}{n!} \]
(d) \[ \sum_{n=1}^{\infty} \sin \left( \frac{1}{n} \right) \]
(e) The Test for Divergence is inconclusive for all of the above series.
21. (10 pts) Find \( \int \frac{2x^2 + 2x + 1}{x^2(x^2 + 1)} \, dx. \)
22. (10 pts) Find \( \int \frac{dx}{x^2 \sqrt{x^2 + 4}} \).
23. (10 pts) Determine whether the following series converges or diverges. If it converges, find the sum.

(i) \[ \sum_{n=1}^{\infty} \frac{(-2)^n + 3^n}{10^n} \]

(ii) \[ \sum_{n=1}^{\infty} \frac{4}{n^2 + 2n} \]
24. (10 pts) Suppose \( f(x) = e^{-x} + x, \ 0 \leq x \leq 1 \)

a.) Set up but do not evaluate an integral that gives the surface area obtained by rotating the curve around the \( x \)-axis.

b.) Set up but do not evaluate an integral that gives the surface area obtained by rotating the curve around the \( y \)-axis.