MATH 152, SPRING 2017
COMMON EXAM I - VERSION A

LAST NAME(print): __________________________________________ FIRST NAME(print): __________________________________________

INSTRUCTOR: __________________________________________

SECTION NUMBER: ________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE HONOR CODE

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: __________________________________________
Part 1: Multiple Choice (4 points each)

1. A 20-ft rope weighing 4 lb/ft is hanging off a cliff with a 30 lb weight attached. How much work is required to lift the whole rope and weight to the top of the cliff?
   (a) 800 ft-lb
   (b) 600 ft-lb
   (c) 2200 ft-lb
   (d) 1400 ft-lb
   (e) None of these

2. Compute $\int_{1}^{2} x^2 e^{x^3} \, dx$.
   (a) $\frac{1}{3} (e^8 - e)$
   (b) $\frac{1}{3} (e^2 - e)$
   (c) $3 (e^8 - e)$
   (d) $3 (e^2 - e)$
   (e) None of these

3. The region bounded by the curves $y = x^2$ and $y = 1$ is rotated about the line $y = 1$. Find the volume of the resulting solid.
   (a) $\frac{8\pi}{5}$
   (b) $\frac{16\pi}{15}$
   (c) $\frac{4\pi}{3}$
   (d) $\frac{12\pi}{5}$
   (e) None of these
4. Compute \( \int_1^e x^2 \ln x \, dx \).

- (a) \( \frac{2}{9} e^3 - \frac{1}{9} \)
- (b) \( 1 - e \)
- (c) \( \frac{2}{9} e^3 + \frac{1}{9} \)
- (d) \( e^2 - \frac{1}{9} e^3 + \frac{1}{9} \)
- (e) None of these

5. Find the average value of the function \( f(x) = \sqrt{4x + 1} \) on the interval \([0, 2]\).

- (a) \( \frac{13}{3} \)
- (b) \( \frac{26}{3} \)
- (c) \( \frac{52}{3} \)
- (d) \( \frac{104}{3} \)
- (e) None of these

6. Compute \( \int 6 \cos^2(4x) \, dx \).

- (a) \( 3x - \frac{3}{8} \sin 8x + C \)
- (b) \( 3x + \frac{3}{8} \sin 8x + C \)
- (c) \( \frac{1}{2} \cos^3(4x) + C \)
- (d) \( 3x + 3 \sin 8x + C \)
- (e) \( 3x - 3 \sin 8x + C \)
7. Which of the following gives the total area between the curves \( y = \cos x \) and \( y = \sin x \) on the interval from \( x = 0 \) to \( x = \frac{\pi}{2} \)?

(a) \( \int_{0}^{\pi/4} (\cos x - \sin x) \, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x) \, dx \)

(b) \( \int_{0}^{\pi/4} (\sin x - \cos x) \, dx + \int_{\pi/4}^{\pi/2} (\cos x - \sin x) \, dx \)

(c) \( \int_{0}^{\pi/2} (\cos x - \sin x) \, dx \)

(d) \( \int_{0}^{\pi/2} (\sin x - \cos x) \, dx \)

(e) None of these

8. The region bounded by the curves \( y = \frac{1}{x} \), \( y = 1 \), \( y = 3 \), and the \( y \)-axis is rotated about the line \( x = -2 \). Find the volume of the resulting solid.

(a) \( \pi \left( \frac{26}{3} + 4 \ln 3 \right) \)

(b) \( \frac{2 \pi}{3} \)

(c) \( \pi (4 + 4 \ln 3) \)

(d) \( \pi \left( \frac{2}{3} + 4 \ln 3 \right) \)

(e) None of these
9. Compute $\int \cos^4 x \sin^5 x \, dx$.

(a) $-\frac{1}{5} \cos^5 x + \frac{1}{9} \cos^9 x + C$

(b) $\frac{1}{6} \sin^6 x - \frac{1}{4} \sin^8 x + \frac{1}{10} \sin^{10} x + C$

(c) $\frac{1}{6} \sin^6 x - \frac{1}{10} \sin^{10} x + C$

(d) $-\frac{1}{5} \cos^5 x + \frac{2}{7} \cos^7 x - \frac{1}{9} \cos^9 x + C$

(e) None of these

10. The region bounded in the first quadrant by the curves $y = 3 - x^2$, $y = 2x$ and the $x$-axis is rotated about the line $y = 5$. Which of the following gives the volume of the resulting solid?

(a) $\int_0^1 2\pi \left( \sqrt{3 - y} - \frac{1}{2}y \right)(5 - y) \, dy$

(b) $\int_0^2 2\pi \left( \sqrt{3 - y} - \frac{1}{2}y \right)(5 - y) \, dy$

(c) $\int_0^2 2\pi \left( \sqrt{3 - y} - \frac{1}{2}y \right)(y - 5) \, dy$

(d) $\int_0^1 2\pi \left( \sqrt{3 - y} - \frac{1}{2}y \right)(y - 5) \, dy$

(e) None of these
11. Compute \( \int_0^1 \frac{x+4}{x^2+1} \, dx \).

(a) \( \frac{1}{2} \ln 2 + \pi \)
(b) \( 4 \ln 2 + \pi \)
(c) \( 4 \ln 2 \)
(d) \( \frac{1}{2} \ln 2 \)
(e) None of these

12. The region bounded by the curves \( y = e^x \), \( x = 0 \), \( x = 2 \) and the \( x \)-axis is rotated about the \( y \)-axis. Find the volume of the resulting solid.

(a) \( 2\pi (e^2 + 1) \)
(b) \( 4\pi e^2 \)
(c) \( \frac{\pi}{2} (e^4 - 1) \)
(d) \( \pi (e^4 - 1) \)
(e) None of these

13. Which of the following gives the area of the region bounded by the curves \( x = y^2 \) and \( x + y = 6 \).

(a) \( \int_{-3}^{2} (y^2 - 6 + y) \, dy \)
(b) \( \int_{4}^{9} (6 - x - \sqrt{x}) \, dx \)
(c) \( \int_{-3}^{2} (6 - y - y^2) \, dy \)
(d) \( \int_{4}^{9} (\sqrt{x} - 6 + x) \, dx \)
(e) None of these
14. Compute \( \int \tan^2 x \sec^4 x \, dx \).

(a) \( \frac{1}{5} \tan^5 x - \frac{1}{3} \tan^3 x + C \)
(b) \( \frac{1}{3} \tan^3 x + C \)
(c) \( \frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C \)
(d) \( \frac{1}{7} \sec^7 x + \frac{1}{5} \sec^5 x + C \)
(e) \( \frac{1}{5} \tan^5 x + \frac{1}{3} \tan^3 x + C \)

15. The work required to stretch a spring from its natural length to 4 m beyond its natural length is 40 J. How much work is required to stretch the spring from its natural length to 5 m beyond its natural length?

(a) 125 J
(b) 20 J
(c) 50 J
(d) \( \frac{125}{2} \) J
(e) None of these

Part 2: Work Out

**Directions:** Present your solutions in the space provided. Show all your work neatly and concisely and box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (8 points) Compute \( \int x^3(x^2 + 1)^7 \, dx \).
17. (6 points) Compute \( \int x^2 \sin(3x) \, dx \).

18. (8 points) Find the volume of the solid whose base is the ellipse \( x^2 + 4y^2 = 4 \) and whose cross-sections perpendicular to the \( y \)-axis are squares. Evaluate your integral.
19. (10 points) Consider the region bounded by the curves $y = 5x - x^2$ and $y = 5 - x$.

(a) Set up an integral to find the volume of the solid formed by rotating this region about the line $y = 8$. Do not evaluate your integral.

(b) Set up an integral to find the volume of the solid formed by rotating this region about the line $x = -3$. Do not evaluate your integral.
20. (8 points) A tank filled with water is in the shape of a trough with isosceles triangles as its ends. (See picture below). The trough is 20 m long, has a height of 6 m, and the width of the trough across the top is 3 m. The trough has a spout with height 1 m. Note: The weight density of water is $\rho g = 9800 \text{ N/m}^3$. Set up an integral to find the work required to pump all the water out of the tank. Do not evaluate your integral. Clearly indicate in the picture below where you are placing your axis and which direction is positive.