MATH 152, Fall 2017
COMMON EXAM I - VERSION A

LAST NAME(print): ___________ KEY ___________ FIRST NAME(print): ___________

INSTRUCTOR: ___________

SECTION NUMBER: ___________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-19), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 20-22), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________
Part 1: Multiple Choice (4 points each)

1. A spring has a natural length of 2 m. If a force of 24 N is required to stretch the spring to a length of 6 m, how much work would be required to stretch the spring from 3 to 5 m?

   (a) 12 Nm
   (b) 16 Nm
   (c) 24 Nm
   (d) 48 Nm
   (e) 50 Nm

\[ F = 24 \text{ N} \quad \Rightarrow \quad k \Delta x = k(6 - 2) = 4. \]

\[ F \Delta x = \int_{3}^{5} kx \, dx = \int_{3}^{5} 6x \, dx = \left[ 3x^2 \right]_{3}^{5} = 3(9 - 1) = 24 \text{ Nm} \]

2. Evaluate the indefinite integral \( \int x^3 e^{x^2} \, dx \).

   (a) \( C + x^2 e^{x^2} - e^{x^2} \)
   (b) \( C + \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} \)
   (c) \( C + x^2 e^{x^2} - 2xe^{x^2} + e^{x^2} \)
   (d) \( C + \frac{1}{3} x^2 e^{x^2} - \frac{1}{3} xe^{x^2} + e^{x^2} \)
   (e) \( C + x^2 e^{x^2} + 2xe^{x^2} + 3e^{x^2} \)

Let \( n = x^2 \), then \( dn = 2x \, dx \)

\[ \int x^3 e^{x^2} \, dx = \int x^3 e^n \, dn = \frac{1}{2} \int e^n \, dn \]

Now Integration by parts:

\[ u = n \quad \Rightarrow \quad du = dn \]
\[ dv = e^n \, dn \quad \Rightarrow \quad v = e^n \]

\[ \frac{1}{2} \int e^n \, dn = \frac{1}{2} \left[ ne^n - \int e^n \, dn \right] + c \]

\[ \frac{1}{2} \int x^3 e^{x^2} \, dx = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + c \]

3. The region bounded by the curves \( y = x^2 \) and \( y = x \) is rotated about the \( x \)-axis. Find the volume of the resulting solid.

   (a) \( \frac{\pi}{3} \)
   (b) \( \frac{2\pi}{5} \)
   (c) \( \frac{\pi}{15} \)
   (d) \( \frac{\pi}{1} \)
   (e) \( \frac{2\pi}{15} \)

\[
\text{Washers: } V = \pi \int_{0}^{1} (R^2 - r^2) \, dx \\
R = y = x \\
r = y = x^2 \\
V = \pi \int_{0}^{1} (x^2 - (x^2)^2) \, dx \\
= \pi \int_{0}^{1} (x^2 - x^4) \, dx \\
= \pi \left[ \frac{x^3}{3} - \frac{x^5}{5} \right]_{0}^{1} = \pi \left( \frac{1}{3} - \frac{1}{5} \right) = \pi \left( \frac{2}{15} \right) 
\]
4. Compute \( \int_1^e \frac{\ln x (1 + \ln x)}{x} \, dx \).

(a) \( \frac{1}{6} \)
(b) \( \frac{e^3}{3} - \frac{e^2}{2} + \frac{1}{6} \)
(c) \( \frac{5}{6} \)
(d) \( \frac{e^3}{3} + \frac{e^2}{2} - \frac{5}{6} \)
(e) \( \frac{23}{6} \)

\[
\int_1^e \frac{\ln x (1 + \ln x)}{x} \, dx = \int_1^1 \frac{\ln x}{x} \, dx + \int_1^1 \frac{(\ln x)^2}{x} \, dx.
\]

\[
\begin{align*}
\theta & \quad x = 1, \quad u = \ln x \quad du = \frac{1}{x} \, dx \\
\theta & \quad x = e, \quad u = \ln (e) = 1
\end{align*}
\]

\[
= \left[ \frac{1}{2} u^2 \right]_1^e + \left[ \frac{1}{3} u^3 \right]_1^e = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}
\]

5. 50 ft of rope weighing 200 lb hangs from the top of a well with a bucket attached to it. The bucket is used to draw water from the well. When the bucket is full of water, it weighs 25 lb. How much work would be required to pull up 10 ft of rope when the bucket is full of water?

(a) 1800 ft-lb
(b) 2050 ft-lb
(c) 32000 ft-lb
(d) 1150 ft-lb
(e) 40000 ft-lb

\[
50 \text{ ft of rope weighs } 200 \text{ lb}
\]

\[
\begin{align*}
50 \quad \text{ft weights } & \quad 200 \quad \text{lb} \\
\quad \text{g} & \quad \frac{200}{50} = 4 \quad \text{lb/ft}
\end{align*}
\]

\[
W = \int_{40}^{50} 4 y \, dy + (25)(10)
\]

\[
W = \left[ 4 \frac{y^2}{2} \right]_{40}^{50} + 250 = 2(500 - 1600) + 250 = 2050 \quad \text{ft-lb}
\]

6. Compute \( \int_0^1 \frac{1}{(2x - 3)^2} \, dx \).

(a) \( \frac{1}{3} \)
(b) \( -\frac{1}{3} \)
(c) \( \frac{2}{3} \)
(d) \( \frac{1}{6} \)
(e) DNE

\[
\begin{align*}
\int_0^1 \frac{1}{(2x - 3)^2} \, dx & = \int_0^1 \frac{1}{u^2} \, du = \frac{1}{2} \int u^{-2} \, du = -\frac{1}{2u}
\end{align*}
\]

\[
= -\frac{1}{2} \left[ \frac{1}{(2x-3)} - \frac{1}{(0-3)} \right] = -\frac{1}{2} \left[ -1 + \frac{1}{3} \right]
\]

\[
= -\frac{1}{2} \left( -\frac{2}{3} \right) = \frac{1}{3}
\]
7. Find the area of the region bounded by \( y = \cos(x) \), \( y = -1 \), \( x = 0 \) and \( x = \frac{\pi}{2} \).
(a) \(-1 + \frac{\pi}{2}\)
(b) \(1 + \frac{\pi}{2}\)
(c) \(\frac{\pi}{2}\)
(d) \(\frac{\pi}{2} + 2\)
(e) \(\pi\)
\[
A = \int \left[ (1 - \cos(x)) \right] dx
= \left[ \sin(x) \right]_0^{\pi/2}
= (1 - 0) + (\pi/2 - 0)
= 1 + \pi/2
\]

8. Which of the following integrals accurately represents the volume of the solid obtained by rotating the region bounded by the curves \( y = 8x^3 \), \( y = 0 \) and \( x = 1 \) about the line \( x = 2 \)?
(a) \(\int_0^1 2\pi(2 - x)(8x^3)dx\)
(b) \(\int_0^1 2\pi x(8x^3 - 2)dx\)
(c) \(\int_1^2 2\pi(x - 2)(8x^3 - 2)dx\)
(d) \(\int_1^2 2\pi(x + 2)(8x^3 - 2)dx\)
(e) \(\int_0^1 2\pi(2 + x)(8x^3)dx\)

But also, shells \(\rightarrow\) disks \(\rightarrow\)
\[V = 2\pi \int_0^1 (2-x)(8x^3 - 0) dx\]

9. Compute \(\int \cos^3(x) \sqrt{\sin(x)} \, dx\).
(a) \(C + \frac{2}{7}(\sin(x))^{3/2} - \frac{2}{3}(\sin(x))^{3/2}\)
(b) \(C + \frac{2}{7}(\sin(x))^{7/2} + \frac{2}{3}(\sin(x))^{3/2}\)
(c) \(C + \frac{2}{3}(\sin(x))^{-1/2}(\cos(x))^2\)
(d) \(C - \frac{2}{3}(\sin(x))^{3/2} + \frac{2}{7}(\cos(x))^{7/2}\)
(e) \(C + \frac{2}{3}(\sin(x))^{3/2} - \frac{2}{7}(\sin(x))^{7/2}\)

\[\sin(x)\text{ is odd, } \cos(x)\text{ is odd}\]
\[\text{But } \sin(x) \text{ under square root}\]
\[u = \sin(x)\]
\[du = \cos(x) \, dx\]
\[= \int \cos^2(x) \sqrt{\sin(x)} \cos(x) \, dx\]
\[= \int (\sin(x) - u^2) \sqrt{u} \, du = \int u^{3/2} - u^{5/2} \, du\]
\[\left[ u^{3/2} - u^{5/2} \right]^{3/2}_{5/2} + C\]
\[= \frac{u^{3/2}}{3/2} - \frac{u^{5/2}}{5/2} + C\]
10. Compute \( \int_a^b 4 + 4x \frac{dx}{1 + x^2} \).

(a) 4 \ln(2) 
(b) \pi + 2 \ln(2) 
(c) 2 \ln(2) 
(d) \frac{\pi}{4} + 4 \ln(4) 
(e) \pi + 4 \ln(2) 

\[
\begin{align*}
\int_0^1 \frac{4}{1 + x^2} \, dx &= \int_0^1 \frac{4x}{1 + x^2} \, dx \\
&= 4 \arctan(x) \Big|_0^1 \\
&= 4 \left( \arctan(1) - \arctan(0) \right) \\
&= 4 \left( \frac{\pi}{4} - 0 \right) \\
&= 4 \left( \frac{\pi}{4} \right) \\
&= 2 \ln(1 + x^2) \Big|_0^1 \\
&= 2 \ln(2) - 2 \ln(1).
\end{align*}
\]

\[\pi + 2 \ln(2)\]

11. Compute \( \int_0^1 x^3(x^2 - 1)^4 \, dx \).

(a) \frac{1}{30} 
(b) -\frac{1}{30} 
(c) -\frac{1}{15} 
(d) \frac{1}{60} 
(e) -\frac{1}{60} 

\[
\begin{align*}
&u = x^2 - 1 \quad \Rightarrow \quad x^2 = u + 1 \\
&du = 2x \, dx \\
&\int x^3(x^2 - 1)^4 \, dx = \int (u^5 + u^4) \, dx = \frac{1}{2} \int (u^5 + u^4) \, du \\
&= \frac{1}{2} \left[ \frac{u^6}{6} + \frac{u^5}{5} \right] = \frac{1}{2} \left[ \frac{(x^2 - 1)^6}{6} + \frac{(x^2 - 1)^5}{5} \right] \\
&= \frac{1}{2} \left[ 0 - \frac{1}{6} + 0 - \frac{1}{5} \right] = \frac{1}{2} \left[ -\frac{5}{6} + \frac{6}{5} \right] = \frac{1}{60}
\end{align*}
\]

12. Compute the indefinite integral \( \int 2 \cos^2(x) \, dx \).

(a) \( C + x - \frac{\sin(2x)}{2} \) 
(b) \( C + x + 2 \sin(2x) \) 
(c) \( C + x - \frac{\sin(2x)}{2} \) 
(d) \( C + \frac{x}{2} + \frac{\sin(2x)}{4} \) 
(e) \( C + x + \frac{\sin(2x)}{2} \)

\[
\begin{align*}
\text{double angle formula} \\
\cos^2(x) &= \frac{1 + \cos(2x)}{2} \\
2 \int \cos^2(x) \, dx &= 2 \int \frac{1}{2} \, dx + 2 \int \frac{1}{2} \cos(2x) \, dx \\
&= x + \frac{\sin(2x)}{2} + C
\end{align*}
\]
13. Evaluate the definite integral $\int_1^3 \ln(x) \, dx$.

(a) $3\ln(3) - 2$
(b) $3\ln(3) - 3$
(c) $\frac{\ln(3)}{3} - 1$
(d) $\frac{\ln(3)}{3} - \frac{1}{3}$
(e) $-\frac{1}{3}$

\[ \int \ln(x) \, dx \rightarrow \text{Integration by parts} \]
\[ u = \ln(x) \quad \quad \quad dv = dx \]
\[ du = \frac{1}{x} \, dx \quad \quad \quad v = x \]

\[ \int \ln(x) \, dx = x \ln(x) - \int x \cdot \frac{1}{x} \, dx \]
\[ = x \ln(x) - x \quad \int \ln(x) \, dx = x \ln(x) - 3 \]
\[ = 3 \ln(3) - 1 \ln(1) - (3 - 1) \]
\[ = 3 \ln(3) - 2 \]

14. Compute $\int x^3 \sin(x) \, dx$.

(a) $C + x^3 \cos x + 3x^2 \cos x - 6x \sin x + 6 \cos x$
(b) $C + x^3 \cos x + 3x^2 \sin x - 6x \cos x + 6 \sin x$
(c) $C - x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x$
(d) $C - x^3 \cos x - 3x^2 \sin x + 6x \cos x + 6 \sin x$
(e) $C + \frac{1}{4} x^4 - \frac{1}{3} x^3 \sin x + \frac{1}{6} x^2 \cos x + \frac{1}{6} x \sin x$

\[ \int x^3 \sin(x) \, dx = -x^3 \cos(x) + 3x^2 \sin(x) - 6x \cos(x) - 6 \sin(x) + C \]

15. A solid has a base that is bounded by the curves $y = 4 - x^2$ and the line $y = 0$. Cross sections orthogonal to the base and perpendicular to the $y-$axis are squares. What is the volume of the solid?

(a) 8
(b) 16
(c) $\frac{16}{3}$
(d) 32
(e) $\frac{32}{3}$

The area of the base is $A = (2x)^2 = 4x^2$

\[ V = \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dy \, dx = 4 \int_0^4 (4-y) \, dy = 32 \]
16. Which of the following integrals accurately represents the volume of the solid formed by rotating the region bounded by the curves $y = 6x - x^2$ and $x$-axis about the $x$-axis?

(a) $\int_{0}^{6} \pi (6x - x^2)\,dx$
(b) $\int_{0}^{6} \pi (6x - x^2)^2\,dx$
(c) $\int_{0}^{6} 2\pi x (6x - x^2)\,dx$
(d) $\int_{0}^{6} 2\pi y (6y - y^2)\,dy$
(e) $\int_{0}^{6} \pi (6y - y^2)^2\,dy$

17. Which of the following integrals accurately represents the volume of the solid formed by rotating the region bounded by the curves $y = 6x - x^2$ and $x$-axis about the line $x = -1$?

(a) $\int_{0}^{6} \pi ((6x - x^2 - 1)^2 - 1)\,dx$
(b) $\int_{0}^{6} 2\pi (6x - x^2)(x+1)\,dx$
(c) $\int_{0}^{6} 2\pi (6x - x^2)(x)\,dx$
(d) $\int_{0}^{6} \pi ((6y - y^2 + 1)^2 - 1)\,dy$
(e) $\int_{0}^{6} \pi (6x - x^2)(y+1)\,dy$

18. A spherical tank with a radius of 1 m is half filled with water. The tank has a spout 2 m high at its topmost point. Find the work required to pump all the water out of the tank from the spout at the top of the tank.

(a) $\frac{17}{4} \pi pg$ Nm
(b) $\frac{7}{4} \pi pg$ Nm
(c) $\frac{15}{4} \pi pg$ Nm
(d) $\frac{9}{4} \pi pg$ Nm
(e) $\frac{17}{2} \pi pg$ Nm
19. Find the area between the curves $y = x^2 - 2$, $y = x$, $x = 0$ and $x = 3$.

(a) $\frac{31}{6}$
(b) $\frac{5}{2}$
(c) $\frac{55}{6}$
(d) $\frac{3}{2}$
(e) $\frac{10}{3}$

**Part 2: Work Out**

Directions: Present your solutions in the space provided. *Show all your work neatly and concisely and box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

20. (8 points) Compute $\int \tan^2(x) \sec^5(x) \, dx$.

\[
even \sec(x) \quad \Rightarrow \quad u = \tan(x) \\
du = \sec^2(x) \, dx
\]

\[
= \int \tan^2(x) \cdot \sec^4(x) \cdot \sec(x) \, dx
\]

Also $\sec^2(x) = 1 + \tan^2(x) \rightarrow 1 + u^2$

\[
= \int u^2 \cdot (1+u^2)^2 \, du
\]

\[
= \int u^2 \cdot (1 + 2u^2 + u^4) \, du
\]

\[
= \int u^2 + 2u^4 + u^6 \, du
\]

\[
= \frac{u^3}{3} + \frac{2u^5}{5} + \frac{u^7}{7} + C
\]

\[
= \frac{1}{3} (\tan(x))^3 + \frac{2}{5} (\tan(x))^5 + \frac{1}{7} (\tan(x))^7 + C
\]

For very B

\[
\int \tan^5(x) \sec^3(x) \, dx
\]

\[
= \frac{1}{3} \sec(x)^3 - \frac{2}{5} \sec(x)^5 + \frac{1}{3} \sec(x)^3 + C
\]
21. (8 points) Find the area of the region enclosed by the function $y = 2x^3$, the line $y = 0$ and the line tangent to the function at the point $(1, 2).

\[ y = 2x^3 \quad \delta \quad x = \left( \frac{1}{6} \right)^{1/3} \]

\[ y = 2 \]

\[ (\frac{2}{3}, 0) \quad x = 1 \]

\[ y = 2x^3 \quad \text{slope of tangent line} \]

\[ m = y' = 6x \quad \bigg|_{(1, 2)} \]

\[ = 6(1) = 6 \]

\[ y - 2 = 6(x - 1) \]

\[ x = \frac{y+4}{6} \]

\[ \Rightarrow y = 6x - 4 \quad \text{equation of tangent line} \]

\[ @ y = 0, \quad x = \frac{4}{6} = \frac{2}{3} \]

\[ A = \int_{0}^{2/3} (2x^3 - 0) \, dx + \int_{2/3}^{1} (2x^3 - (6x-4)) \, dx \]

\[ \text{In } (y) \rightarrow R-L \]

\[ A = 2 \int_{0}^{1} \left[ \left( \frac{y+4}{6} \right) - \frac{y^{\frac{1}{3}}}{2} \right] \, dy \]

Area of enclosed region: $\frac{1}{6}$.

\[ \text{For } \overline{BC} \quad \overline{2/3}, \quad \text{tangent line is } \quad y = 9x - 6 \quad @ y = 0, \quad x = \frac{2}{3} \]

\[ A = \int_{0}^{2/3} (3x^3 - 0) \, dx + \int_{2/3}^{1} (3x^3 - (9x-6)) \, dx \]

or \[ A = 3 \int_{0}^{\frac{2}{3}} \left( \frac{y+6}{9} \right) - \left( \frac{y^{\frac{1}{3}}}{3} \right)^{2/3} \, dy \]

\[ \text{Area} = \frac{1}{4} \]
22. (8 points) Evaluate \( \int 5e^{2x} \cos(2x) \, dx \) = \( Q \).

\[
U = e^{2x} \\
\frac{du}{dx} = 2e^{2x} \\

\frac{dv}{dx} = \cos(2x) \\
v = \frac{\sin(2x)}{2}
\]

\[
5 \int e^{2x} \cos(2x) \, dx = 5 \left[ \frac{e^{2x} \sin(2x)}{2} - \int \frac{\sin(2x)}{2} \cdot 2e^{2x} \, dx \right]
\]

for \( \int e^{2x} \sin(2x) \, dx \):

\[
U = e^{2x} \\
\frac{du}{dx} = 2e^{2x} \\

\frac{dv}{dx} = \sin(2x) \\
v = -\cos(2x)
\]

\[
\int e^{2x} \sin(2x) \, dx = -\frac{e^{2x} \cos(2x)}{2} - \int -\cos(2x) \cdot 2e^{2x} \, dx
\]

\[
5 \int e^{2x} \cos(2x) \, dx = 5 \frac{e^{2x} \sin(2x)}{2} - 5 \left[ -\frac{e^{2x} \cos(2x)}{2} \right] + 5 \left( \int e^{2x} \cos(2x) \, dx \right)
\]

\[
5 \int e^{2x} \cos(2x) \, dx = \frac{5}{2} e^{2x} \sin(2x) + \frac{5}{2} e^{2x} \cos(2x) - 5 \int e^{2x} \cos(2x) \, dx
\]

\[
\therefore 2Q = \frac{5}{2} e^{2x} (\sin(2x) + \cos(2x))
\]

\[
\therefore Q = \frac{5}{4} e^{2x} (\sin(2x) + \cos(2x)) + C
\]

For part B

\[
\int 3e^{2x} \sin(2x) \, dx = \frac{3}{4} e^{2x} (-\cos(2x) + \sin(2x)) + C
\]

FOR INSTRUCTOR USE ONLY

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