MATH 152, Fall 2017
COMMON EXAM III - VERSION B

LAST NAME(print): KEY FIRST NAME(print): ________________________

INSTRUCTOR: ________________________

SECTION NUMBER: ____________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-16), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 17-19), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ________________________
Part 1: Multiple Choice (5 points each)

1. Which of the following is true regarding the series \( \sum_{n=1}^{\infty} \frac{5n \cdot 2^{2n}}{3^n} \)?
   
   (a) The Ratio test limit is 4/3, so the series diverges.
   (b) The Ratio test limit is 4/3, so the series converges.
   (c) The Ratio test limit is 2/3, so the series converges.
   (d) The Ratio test limit is 2/3, so the series diverges.
   (e) The Ratio test limit is 20/3, so the series diverges.

2. The series \( \sum_{i=1}^{\infty} \left( e^{2/i} - e^{2/(i+1)} \right) \)
   
   (a) Converges to e.
   (b) Converges to e - \( e \).
   (c) Converges to \( e^2 \).
   (d) Converges to 0.
   (e) Diverges.

3. Which of the following statements are true regarding the pair of infinite series below?
   
   \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{3/2}} \) \hspace{1cm} \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{2/5}} \)
   
   (a) Both series are divergent.
   (b) Both series are absolutely convergent.
   (c) (I) is absolutely convergent; (II) is convergent, but not absolutely convergent.
   (d) (II) is absolutely convergent; (I) is convergent, but not absolutely convergent.
   (e) (I) is convergent, but not absolutely convergent; (II) is convergent, but not absolutely convergent.
4. The recursive sequence given below is bounded and decreasing. Find the limit of the sequence, if it exists.
\[ a_1 = 5, \quad a_{n+1} = \frac{28}{11 - 3n} \]

(a) 0
(b) \( \frac{28}{11} \)
(c) -17
(d) 4
(e) 7

5. Which of the following statements is true regarding the sequence \( a_n = \ln(5n + 2) - \ln(3n - 1) \)?

(a) The sequence converges to \( \frac{5}{3} \).
(b) The sequence converges to 0.
(c) The sequence converges to \( \ln \left( \frac{5}{3} \right) \).
(d) The sequence diverges to \( \infty \).
(e) The sequence diverges to \( -\infty \).
6. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{3n}}{(-5)^n}$ if it exists.

(a) The series diverges.
(b) $-4$
(c) $-\frac{2}{7}$
(d) $-\frac{4}{9}$
(e) $\frac{4}{9}$

7. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$

(II) $\sum_{n=2}^{\infty} (-1)^n \left(\frac{4}{3}\right)^n$

(III) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

(a) Only I and III converge.
(b) Only II and III converge.
(c) Only I and II converge.
(d) Only I converges.
(e) All three series converge.

8. Find the sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)}$. (Hint: Partial fractions)

(a) $\frac{1}{4}$
(b) $\frac{1}{5}$
(c) $12$
(d) $\frac{1}{3}$
(e) $1$
9. Given a series \( \sum_{n=1}^{\infty} a_n \) whose \( n \)-th partial sum is given by \( S_n = \frac{3n^2 + 4}{5 - 4n^2} \), which statement is true about the series?

(a) The series converges to \( \frac{3}{4} \).
(b) The series converges to \( -\frac{3}{4} \).
(c) The series converges to \( \frac{4}{5} \).
(d) The series converges to \( \frac{3}{5} \).
(e) The series diverges.

10. Identify the conic section \( x^2 + 16y^2 = 16 \) and find its focus point(s).

(a) Hyperbola with focus points \((0, \pm \sqrt{17})\).
(b) Hyperbola with focus points \((\pm \sqrt{17}, 0)\).
(c) Parabola with focus point \((0, 1)\).
(d) Ellipse with focus points \((0, \pm \sqrt{15})\).
(e) Ellipse with focus points \((\pm \sqrt{15}, 0)\).

11. Which of the following series converge?

(I) \( \sum_{n=1}^{\infty} \frac{(-3)^n}{n!} \)  
(II) \( \sum_{n=1}^{\infty} \frac{ne^{-n}}{n!} \)  
(III) \( \sum_{n=1}^{\infty} \frac{n^2}{n!} \)

(a) Only I and III converge.
(b) Only II and III converge.
(c) Only I and II converge.
(d) Only I converges.
(e) All three series converge.
12. Using the error estimate in the Alternating Series Test, find the minimum number of terms required to approximate the sum of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \) with an accuracy less than or equal to \( \frac{1}{100} \).

(a) 8 terms.
(b) 7 terms.
(c) 5 terms.
(d) 4 terms.
(e) 3 terms.

13. Which of the following statements is true regarding the sequence \( a_n = 3 + e^{1/n} \)?

(a) \( a_n \) is decreasing and converges to 4.
(b) \( a_n \) is increasing but divergent.
(c) \( a_n \) is increasing and converges to 4.
(d) \( a_n \) is decreasing but divergent.
(e) \( a_n \) is increasing and converges to 3.
14. If \( b_n \geq a_n \geq 0 \) is true for every positive integer \( n \), which of the following statements is also always true?

(a) If \( \lim_{n \to \infty} b_n = 0 \), then \( \sum_{n=1}^{\infty} a_n \) is convergent.
(b) If \( \lim_{n \to \infty} a_n = 0 \), then \( \lim_{n \to \infty} b_n = 0 \).
(c) If \( \sum_{n=1}^{\infty} a_n \) is divergent, then so is \( \sum_{n=1}^{\infty} b_n \).
(d) If \( \sum_{n=1}^{\infty} b_n \) is divergent, then so is \( \sum_{n=1}^{\infty} a_n \).
(e) If \( \sum_{n=1}^{\infty} a_n \) is convergent, then so is \( \sum_{n=1}^{\infty} b_n \).

15. Consider the infinite series \( \sum_{n=1}^{\infty} \cos \left( \frac{1}{n^2} \right) \). Which of the following statements is true?

(a) The Test for Divergence shows that the series is divergent.
(b) The Comparison Test shows that the series is convergent.
(c) The Ratio Test shows that the series diverges.
(d) The Limit Comparison Test shows that the series converges.
(e) The Ratio Test shows that the series converges.

16. Which of the statements below is true regarding the sequence \( a_n = \frac{(-1)^n(n + 3)}{5n - 3} \)?

(a) The sequence is bounded and converges to 0.
(b) The sequence is bounded, but diverges.
(c) The sequence is not bounded and diverges.
(d) The sequence is not bounded, but converges to 1/5.
(e) The sequence is bounded and converges to 1/5.
Part 2: Work Out

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (6 points) Using the Remainder estimate for the Integral Test, find an upper bound on the error in using \( S_9 \) to approximate the sum of the series \( \sum_{n=1}^{\infty} \frac{n}{(n^2 + 1)^5} \).

\[
R_n \leq \int_n^{\infty} f(x) \, dx
\]

\[
\leq \int_n^{\infty} \frac{x}{(x^2 + 1)^5} \, dx
\]

\[
= \frac{1}{2} \int_n^{\infty} \frac{du}{u^5} = \left( -\frac{1}{4u^4} \right) \bigg|_n^{\infty}
\]

\[
\leq -\frac{1}{8(n^2 + 1)^4} \bigg|_3^\infty
\]

\[
R_n \leq -\frac{1}{8} + \frac{1}{8(10)^4}
\]

\[
R_n \leq \frac{1}{8000}
\]
18. Show that the following series converge or prove that they diverge.

- (4 pts) \( \sum_{n=1}^{\infty} \frac{n^3 - n + 7}{n^5 + n^3 + 2} \)
  
  Compare to \( \sum_{n=1}^{\infty} \frac{n^3}{n^5} = \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow \text{converges by p series.} \)

  Since \( \frac{1}{n^2} > \frac{n^3 - n + 7}{n^5 + n^3 + 2} \) (see ver A)

  Series converges by comparison test.

  Let: \( \lim_{n \to \infty} \frac{n^3 - n + 7}{n^5 + n^3 + 2} = \frac{n^3/n^5}{1/n^2} = 1 \)

  1. Series also converges by Limit Comparison test.

- (4 pts) \( \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5} - 1} \)

  Compare to \( \sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}} \rightarrow \text{diverges by p series.} \)

  But \( \frac{1}{\sqrt{n}} > \frac{n^2}{\sqrt{n^5} + 1} \)

  Let: \( \lim_{n \to \infty} \frac{n^2}{\sqrt{n^5} + 1} = \frac{1}{\sqrt{n}} \)

  1. Series diverges by Limit Comparison test.
19. (6 points) Does the series \( \sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n} \) diverge, converge absolutely or converge but not converge absolutely? Fully justify your answer.

a) Alternating series

\[ \sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n} \]

\[ \lim_{n \to \infty} |a_n| = \lim_{n \to \infty} \frac{\ln(n)}{n} = 0 \]

Using L'Hôpital's rule: \( \lim_{n \to \infty} \frac{1}{n} = \frac{1}{\infty} \to 0 \).

ii) \( \frac{d}{dn} \left( \frac{\ln(n)}{n} \right) = \frac{n \cdot \frac{1}{n} - \ln(n)}{n^2} = \frac{1 - \ln(n)}{n^2} \)

decreasing for all \( n > e \) i.e. \( n > 2 \).

i) Alternating series converges.

b) \( \sum_{n=2}^{\infty} |a_n| = \sum_{n=2}^{\infty} \frac{\ln(n)}{n} \)

Integral Test: \( \int_{2}^{\infty} \frac{\ln(x)}{x} \, dx = \left[ \frac{\ln(x)^2}{2} \right]_{2}^{\infty} \to \infty \).

For Instructor Use Only

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