MATH 152 Spring 2018
COMMON EXAM I - VERSION A

LAST NAME: Solutions FIRST NAME: ______________________
INSTRUCTOR: ______________________
SECTION NUMBER: __________
UIN: ______________________

DIRECTIONS:
1. The use of a calculator, laptop or cell phone is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 4 points.
4. In Part 2 (Problems 16-19), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR
"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

DO NOT WRITE BELOW!

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PART I: Multiple Choice. 4 points each.

1. \[ \int_{\pi/2}^{\pi} \cos \left( \frac{1}{3} x \right) \, dx = \]

(a) \[ \frac{3\sqrt{3}}{2} - \frac{3}{2} \]
(b) \[ \frac{3}{2} - 3\sqrt{3} \]
(c) \[ \frac{\sqrt{3}}{6} - \frac{1}{6} \]
(d) \[ \frac{1}{6} - \frac{\sqrt{3}}{6} \]
(e) None of the above

2. The region bounded by \( y = \sqrt{x}, \ y = 0 \) and \( x = 4 \) is rotated around the \( y \) axis. Which of the following integrals gives the resulting volume?

(a) \[ \int_{0}^{2} \pi (4 - y^2)^2 \, dy \]
(b) \[ \int_{0}^{4} \pi (4 - y^2)^2 \, dy \]
(c) \[ \int_{0}^{2} \pi (16 - y^4) \, dy \]
(d) \[ \int_{0}^{4} \pi (16 - y^4) \, dy \]
(e) None of the above

3. A spring has a natural length of 2 m. The force required to keep the spring stretched to a length of 5 m is 10 N. Find the work required to stretch the spring from a length of 3 m to a length of 5 m.

(a) 16 J
(b) \[ \frac{80}{3} \] J
(c) 15 J
(d) 27 J
(e) \[ \frac{40}{3} \] J
4. Which of the following integrals gives the volume of the solid formed by rotating the region bounded by 
\( x = 2y, \ y = 4, \) and \( x = 1 \) around the \( y \)-axis?

\[
\int_1^\infty 2\pi \left( 4 \right) \left( 4 - \frac{x}{2} \right) \, dx
\]

5. \( \int_0^2 xe^{-4x} \, dx = \]

(a) \( \frac{1}{16} - \frac{9}{16e^8} \)
(b) \( \frac{9}{16e^8} \)
(c) \( \frac{24}{e^8} \)
(d) \( \frac{24}{e^8} - 16 \)
(e) \( -\frac{1}{16} + \frac{9}{16e^8} \)

6. \( \int \tan^3 x \sec^3 x \, dx = \]

(a) \( -\frac{1}{5} \sec^5 x + \frac{1}{3} \sec^3 x + C \)
(b) \( -\sec^3 x + \sec^5 x + C \)
(c) \( \frac{3}{5} \tan^3 x - \frac{1}{5} \tan^5 x + C \)
(d) \( \frac{1}{5} \sec^3 x - \frac{1}{3} \sec^5 x + C \)
(e) \( \frac{1}{5} \tan^5 x - \frac{1}{3} \tan^3 x + C \)
7. \[ \int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = \]
(a) \(2e^2 - 2e\)
(b) \(-2e\)
(c) \(2e^2\)
(d) \(\frac{1}{2}e^2 - \frac{1}{2}\)
(e) \(\frac{1}{2}e^2\)

By substitution method, let \(u = \sqrt{x} = x^{1/2}\)
\[ du = \frac{1}{2}x^{-1/2} \, dx \]
so
\[ \int e^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2} \, dx = 2\int e^u \, du \]
\[ = 2e^u \bigg|_{1}^{4} = 2e^4 - 2e \]

Bounds:
\[ u(4) = \sqrt{16} = 2 \]
\[ u(1) = \sqrt{1} = 1 \]

8. The region bounded by \(y = \frac{1}{\sqrt{x}}, x = 1, x = 3\) and \(y = 0\) is revolved around the \(x\) axis. Find the volume.

(a) \(\pi(2\sqrt{3} - 2)\)
(b) \(\pi(\ln 3 - e)\)
(c) \(\pi(\ln 3 - e)\)
(d) \(\pi(\sqrt{3} - 1)\)
(e) None of these

\[ R = \frac{1}{\sqrt{x}} \]

\[ \pi \int_{1}^{3} \left( \frac{1}{\sqrt{x}} \right)^2 \, dx = \pi \int_{1}^{3} \frac{1}{x} \, dx \]
\[ = \pi \left[ \ln |x| \right]_{1}^{3} = \pi \ln (3) - \pi \ln (1) \]
\[ = \pi \ln (3) \quad \text{up} \quad = 0 \]

9. Find the area of the region bounded by the curves \(x = y^2\) and \(x = 9\).

(a) 18
(b) 54
(c) 243
(d) 36
(e) 27

\[ \int_{-3}^{3} (9 - y^2) \, dy \]
\[ 2 \int_{0}^{3} (9 - y^2) \, dy \]
\[ 2 \left[ 9y - \frac{y^3}{3} \right]_{0}^{3} \]
\[ 2 \left[ 9(3) - \frac{27}{3} \right] - 2 \left[ 0 \right] = 2 \left[ 27 - 9 \right] = 2(18) = 36 \]
10. A 25 pound cinder block is attached to a 100 foot rope which is hanging off a tall building. If the rope weighs $\frac{1}{2}$ pounds per foot, find the work done in pulling the rope and cinder block to the top of the building.

(a) 2500 foot pounds
(b) 4000 foot pounds
(c) 2525 foot pounds
(d) 4025 foot pounds
(e) 5000 foot pounds

\[ f(x) = 75 - \frac{1}{2} x \]

\[ W = \int_0^{100} \left( 75 - \frac{1}{2} x \right) \, dx = \left[ 75x - \frac{1}{4}x^2 \right]_0^{100} = \left[ 75(100) - \frac{1}{4}(100)^2 \right] - [0] = 7500 - 2500 = 5000 \]

11. \[ \int x^3 \ln x \, dx = \]

Int. by Parts

\[ u = \ln (x) \quad v = \frac{1}{4} x^4 \]

\[ du = \frac{1}{x} \, dx \quad dv = x^3 \, dx \]

\[ \frac{1}{4} x^4 \ln (x) = \int \frac{1}{4} x^4 \cdot \frac{1}{x} \, dx \]

\[ \frac{1}{4} x^4 \ln (x) - \frac{1}{4} \int x^3 \, dx \]

\[ \frac{1}{4} x^4 \ln (x) - \frac{1}{16} x^4 + C \]

12. \[ \int \cos^2 (3x) \, dx = \]

Double Angle Identity

\[ \int \frac{1}{2} \left[ 1 + \cos (6x) \right] \, dx \]

\[ \frac{1}{2} \left[ x + \frac{1}{6} \sin (6x) \right] + C \]

\[ \frac{1}{2} x + \frac{1}{12} \sin (6x) + C \]
13. Find the area of the region bounded by \( y = x^2 - x \) and \( y = 2x \).

(a) \( \frac{8}{3} \)

(b) \( \frac{9}{2} \)

(c) \( \frac{5}{2} \)

(d) 4

(c) \( \int_0^3 2x - (x^2 - x) \, dx \)

14. Which of the following integrals gives the area of the region bounded by \( y = \cos x \), \( y = 1 - \cos x \), \( 0 \leq x \leq \pi \)?

(a) \( \int_0^{\pi/6} (2 \cos x - 1) \, dx + \int_{\pi/6}^\pi (1 - 2 \cos x) \, dx \)

(b) \( \int_0^{\pi/3} (1 - 2 \cos x) \, dx + \int_{\pi/3}^{\pi/2} (2 \cos x - 1) \, dx \)

(c) \( \int_0^{\pi/3} (2 \cos x - 1) \, dx + \int_{\pi/3}^{\pi/2} (1 - 2 \cos x) \, dx \)

(d) \( \int_0^\pi (1 - 2 \cos x) \, dx \)

(e) \( \int_0^{\pi/3} (1 - 2 \cos x) \, dx + \int_{\pi/3}^{\pi/2} (2 \cos x - 1) \, dx \)

15. \( \int_{-1}^2 \frac{x}{x + 2} \, dx = \)

(a) \( 2 - \ln 4 \)

(b) \( 3 - 2 \ln 4 \)

(c) \( 1 - \ln 4 \)

(d) \( 1 + \ln 4 \)

(e) \( \ln 4 \)
PART II: Work Out: Box your final answer!

16. Consider the region bounded by \( y = 3 - x^2 \) and \( y = -2x \). Set up but do not evaluate an integral that gives the volume of the solid obtained by revolving the specified region about the given line. DO NOT INTEGRATE.

\[ \text{Part a:} \]
\[ x = -2 \]

(a) (5 pts) About the line \( x = -2 \)

<table>
<thead>
<tr>
<th>Shell method</th>
<th>( r = x - (-2) = x + 2 )</th>
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<tbody>
<tr>
<td>( 3-x^2 = -2x )</td>
<td>( h = (3-x^2) - (-2x) = 3 + 2x - x^2 )</td>
</tr>
<tr>
<td>( 0 = x^2 - 2x - 3 )</td>
<td>( V = \int_{-1}^{3} 2\pi (x+2)(3+2x-x^2) , dx )</td>
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<tr>
<td>( 0 = (x-3)(x+1) )</td>
<td>( x = 3, \ x = -1 )</td>
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(b) (5 pts) About the line \( y = 4 \)

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<tr>
<th>Washer method</th>
<th>( R = 4 - (-2x) = 4 + 2x )</th>
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</thead>
<tbody>
<tr>
<td>( r = 4 - (3-x^2) = 1 + x^2 )</td>
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\[ V = \int_{-1}^{3} \pi \left[ (4+2x)^2 - (1+x^2)^2 \right] \, dx \]
17. (10 pts) A circular cone is filled with water up to a height 2 meters. If the radius of the cone is 1 meter and the height of the cone is 3 meters, set up but do not evaluate an integral that will compute the work required to pump all of the water to the top of the tank. **Indicate on the picture where you are placing the axis and which direction is positive.** DO NOT INTEGRATE. Note: The density of water is \( \rho = 1000 \text{ kg/m}^3 \) and the acceleration due to gravity is 9.8 m/s\(^2\).

Slices are circles,

so \( A_i = \pi r_i^2 \)

\[ A_i = \pi \left( \frac{y}{3} \right)^2 = \frac{\pi}{9} y^2 \]

\( d = 3 - y \)

Water only from \( y = 0 \) to \( y = 2 \), so bounds are \( 0 \) to \( 2 \).

\[ W = \rho g \int_0^2 \left( \frac{\pi}{9} y^2 \right) (3 - y) \, dy \]

\[ W = 9800 \int_0^2 \frac{\pi}{9} y^2 (3 - y) \, dy \]
18. (10 pts) The solid \( S \) has a base in the shape of a triangle with vertices \((0,0)\), \((1,0)\) and \((0,2)\). Cross sections perpendicular to the \( x \)-axis are squares. What is the volume of \( S \)?

Eq. of line: \( m = \frac{2-0}{0-1} = -2 \)

\[ y - 2 = -2(x - 0) \Rightarrow y = -2x + 2 \]

Square cross sections \( \Rightarrow A = s^2 \)

\( S = -2x + 2 \)

\[ V = \int_0^1 (4x^2 - 8x + 4) \, dx \]

\( A = (-2x+2)^2 \)

\[ = \frac{4}{3}x^3 - 4x^2 + 4x \bigg|_0^1 \]

\( A = 4x^2 - 8x + 4 \)

\[ \left[ \frac{4}{3} - 4 + 4 \right] - [0] = \frac{4}{3} \]

19. (10 pts) Consider the region bounded by \( y = x^3 \), the tangent line to this curve at the point \((2, 8)\), and the \( x \)-axis. [See figure below]. Set up but do not evaluate an integral (or integrals) that gives the area of this region. DO NOT INTEGRATE.

\[ y = x^3 \quad y' = 3x^2 \]

@ \( x = 2 \); \( m = 3(2)^2 = 12 \)

Eq. of line: \( y - 8 = 12(x - 2) \)

\[ y = 12x - 16 \]

or

\[ x = \frac{y + 16}{12} \]

In terms of \( x \);

\[ 12x - 16 = 0 \]

\[ 12x = 16 \]

\[ x = \frac{4}{3} \]

\[ \int_0^{4/3} x^3 \, dx + \int_{4/3}^2 \left[ x^3 - (12x - 16) \right] \, dx \]