MATH 152, Fall 2018
COMMON EXAM I - VERSION B

LAST NAME(print): _____________________________ FIRST NAME(print): _____________________________

INSTRUCTOR: __________________________________

SECTION NUMBER: __________________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: _____________________________
Part 1: Multiple Choice (4 points each)

1. Calculate the area of the region bounded by the curves $y = x^2$ and $y = 2 - x^2$.

(a) $\frac{4}{3}$
(b) $\frac{8}{3}$
(c) $4$
(d) $\frac{28}{3}$
(e) None of these

\[ A = \int_{-1}^{1} (2 - x - x^2) \, dx \]
\[ = 2 \int_{0}^{1} (2 - 2x^2) \, dx \]
\[ = 2 \left[ x - \frac{2}{3} x^3 \right]_0^1 \]
\[ = 2 \left( 1 - \frac{2}{3} \right) = \frac{8}{3} \]

2. Compute $\int_0^1 x e^{2x} \, dx$.

(a) $\frac{5}{4} e^2 - \frac{1}{4}$
(b) $e^2 - 2$
(c) $\frac{1}{4} e^2 - \frac{1}{4}$
(d) $5 e^2 - 2$
(e) $10 e^2 - 16$

\[ = \left. \frac{1}{4} e^2 - \frac{1}{4} \right|_0^1 \]

3. Compute $\int_1^4 \frac{\sin(\sqrt{x})}{\sqrt{x}} \, dx$.

(a) $2(-\cos 2 + \cos 1)$
(b) $2(\cos 2 - \cos 1)$
(c) $\frac{1}{2}(\cos 2 - \cos 1)$
(d) $\frac{1}{2}(-\cos 4 + \cos 2)$
(e) $\frac{1}{2}(\cos 4 - \cos 2)$

\[ U = \sqrt{x} \]
\[ du = \frac{1}{2\sqrt{x}} \, dx \]
\[ 2du = \frac{1}{\sqrt{x}} \, dx \]

\[ \int_1^2 2 \sin u \, du = -2 \cos u \bigg|_1^2 \]
\[ = -2 \cos 2 + 2 \cos 1 \]
4. Find the volume of the solid found by rotating the region bounded by the curves \( y = 2x - x^2 \) and \( y = 0 \) about the \( y \)-axis.

(a) \( \frac{16\pi}{3} \)
(b) \( \frac{4\pi}{3} \)
(c) \( \frac{2\pi}{3} \)
(d) \( \frac{8\pi}{3} \)
(e) None of these

\[
\begin{align*}
V &= \int 2\pi rh \, dx \\
&= \int_0^2 2\pi x(2x-x^2) \, dx \\
&= 2\pi \left[ \frac{2}{3}x^3 - \frac{1}{4}x^4 \right]_0^2 \\
&= 2\pi \left( \frac{16}{3} - 4 \right) = 2\pi \left( \frac{4}{3} \right) = \frac{8\pi}{3}
\end{align*}
\]

5. Suppose the work required to stretch a spring from its natural length to 4 m beyond its natural length is 16 J. How much force is needed to hold the spring stretched 6 m beyond its natural length?

(a) 24 N  \hspace{1cm} 16 = \int_0^4 kx \, dx \\
(b) 72 N  \hspace{1cm} f(x) = 2x \\
(c) 36 N  \hspace{1cm} f(6) = 12 N \\
(d) 18 N  \hspace{1cm} 16 = \frac{1}{2}kx^2 \bigg|_0^4 \\
(e) 12 N  \hspace{1cm} 16 = 8k \\
\begin{align*}
2 &= k
\end{align*}

6. Compute \( \int_1^2 x\sqrt{x-1} \, dx \).

(a) 1
(b) \( \frac{16}{15} \)
(c) \( \frac{4}{15} \)
(d) \( \frac{8}{3} \cdot 2^{3/2} - 1 \)
(e) None of these

\[
\begin{align*}
&= \frac{x}{1} \bigg|_0^1 \\
&= \left[ (u+1)^{3/2} + u^{1/2} \right]_0^1 \\
&= \left[ \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} \right]_0^1 \\
&= \frac{2}{5} + \frac{2}{3} = \frac{16}{15}
\end{align*}
\]
7. Which of the following gives the volume of the solid found by rotating the region bounded by the curves \( y = 7 - x^2 \) and \( y = 3 \) about the x-axis?

(a) \( \int_{-2}^{2} 2\pi x (4 - x^2) \, dx \)

(b) \( \int_{-2}^{2} \pi [9 - (7 - x^2)^2] \, dx \)

(c) \( \int_{-2}^{2} \pi (4 - x^2)^2 \, dx \)

(d) \( \int_{-2}^{2} 2\pi x (x^2 - 4) \, dx \)

(\text{Correct Answer}) \( \int_{-2}^{2} \pi [(7 - x^2)^2 - 9] \, dx \)

\[
V = \pi \int_{-2}^{2} (R^2 - r^2) \, dx
\]

\[
= \pi \int_{-2}^{2} [(7 - x^2)^2 - 9] \, dx
\]

\[
= \pi \int_{7 - x^2 = 3}^{7 - x^2 = 4} 7 - x^2 \, dx
\]

\[
= \pi \int_{x^2 = 4}^{x^2 = 9} \sqrt{7 - x^2} \, dx
\]

\[
= \pi \left[ \frac{1}{2} \left( x \sqrt{7 - x^2} + 7 \sin^{-1} \left( \frac{x}{\sqrt{7}} \right) \right) \right]_{x^2 = 4}^{x^2 = 9}
\]

\[
= \pi \left[ \frac{1}{2} \left( x \sqrt{7 - x^2} + 7 \sin^{-1} \left( \frac{x}{\sqrt{7}} \right) \right) \right]_{x = 2}^{x = 3}
\]

\[
= \pi \left[ \frac{1}{2} \left( 3 \sqrt{7 - 9} + 7 \sin^{-1} \left( \frac{3}{\sqrt{7}} \right) \right) \right] - \pi \left[ \frac{1}{2} \left( 2 \sqrt{7 - 4} + 7 \sin^{-1} \left( \frac{2}{\sqrt{7}} \right) \right) \right]
\]

\[
= \pi \left[ \frac{1}{2} \left( \frac{3}{\sqrt{2}} + 7 \sin^{-1} \left( \frac{3}{\sqrt{7}} \right) \right) \right] - \pi \left[ \frac{1}{2} \left( \sqrt{3} + 7 \sin^{-1} \left( \frac{2}{\sqrt{7}} \right) \right) \right]
\]

\[
= \pi \left[ \frac{3}{2\sqrt{2}} + \frac{7}{2} \sin^{-1} \left( \frac{3}{\sqrt{7}} \right) \right] - \pi \left[ \frac{\sqrt{3}}{2} + \frac{7}{2} \sin^{-1} \left( \frac{2}{\sqrt{7}} \right) \right]
\]

\[
= \pi \left[ \frac{3}{2\sqrt{2}} + \frac{7}{2} \sin^{-1} \left( \frac{3}{\sqrt{7}} \right) - \frac{\sqrt{3}}{2} - \frac{7}{2} \sin^{-1} \left( \frac{2}{\sqrt{7}} \right) \right]
\]

8. Which of the following gives the area between the curves \( y = \frac{1}{2}x^3 \) and \( y = 4 \) on the interval \( 0 \leq x \leq 4 \)?

(\text{Correct Answer}) \( \int_{0}^{4} \left( 4 - \frac{1}{2}x^3 \right) \, dx + \int_{4}^{5} \left( 4 - \frac{1}{2}x^3 \right) \, dx \)

\[
= \int_{0}^{4} \left( 4 - \frac{1}{2}x^3 \right) \, dx + \int_{4}^{5} \left( 4 - \frac{1}{2}x^3 \right) \, dx
\]

\[
= \left[ 4x - \frac{1}{6}x^4 \right]_{0}^{4} + \left[ 4x - \frac{1}{6}x^4 \right]_{4}^{5}
\]

\[
= 4(4) - \frac{1}{6}(4)^4 + 4(5) - \frac{1}{6}(5)^4
\]

\[
= 16 - \frac{64}{6} + 20 - \frac{625}{6}
\]

\[
= \frac{96 - 64 + 120 - 625}{6}
\]

\[
= \frac{72}{6}
\]

\[
= 12
\]

9. Compute \( \int \sec^4 x \tan^3 x \, dx \).

(\text{Correct Answer}) \( \frac{1}{7} \sec^7 x + \frac{1}{5} \sec^5 x + C \)

\[
\int \sec^4 x \tan^3 x \, dx = \int \sec^4 x \tan x \tan^2 x \, dx
\]

\[
= \int \sec^4 x (\sec^2 x - 1) \, dx
\]

\[
= \int (\sec^6 x - \sec^4 x) \, dx
\]

\[
= \int \sec^6 x \, dx - \int \sec^4 x \, dx
\]

\[
= \int u^6 \, du - \int \sec^2 x \tan^2 x \, dx
\]

\[
= \int u^6 \, du - \int (u^2 + 1) u^2 \, du
\]

\[
= \int (u^8 + u^6) \, du
\]

\[
= \frac{1}{9} u^9 + \frac{1}{5} u^5 + C
\]

\[
= \frac{1}{7} \tan^7 x + \frac{1}{5} \tan^5 x + C
\]
10. Compute $\int \sin^2(4x) \, dx$.

   \[\int \frac{1}{2} (1 - \cos 8x) \, dx\]
   \[= \frac{1}{2} (x - \frac{1}{8} \sin 8x) + C\]

(a) $\frac{x}{2} + \frac{1}{16} \sin(8x) + C$
(b) $\frac{x}{2} - \frac{1}{16} \sin(8x) + C$
(c) $\frac{x}{2} - \frac{1}{8} \sin(4x) + C$
(d) $\frac{x}{2} + \frac{1}{8} \sin(4x) + C$
(e) $\frac{1}{12} \sin^3(4x) + C$

11. Compute $\int_{1}^{2} \frac{x^3}{\sqrt{x^4 + 1}} \, dx$.

(a) $\frac{1}{2} (\sqrt{2} - 1)$
(b) $8(\sqrt{2} - 1)$
(c) $8(\sqrt{17} - \sqrt{2})$
(d) $\frac{1}{2} (\sqrt{17} - \sqrt{2})$
(e) None of these

\[u = x^4 + 1, \quad \frac{du}{dx} = 4x^3 \quad \text{dx} \]
\[\frac{1}{4} \, du = x^3 \, dx\]
\[\int_{2}^{17} \frac{1}{4u} \, du = \int_{2}^{17} \frac{1}{4} u^{-1/2} \, du\]
\[= \left. \frac{1}{4} \cdot 2u^{1/2} \right|_{2}^{17}\]
\[= \frac{1}{2} (\sqrt{17} - \sqrt{2})\]

12. An 8 ft rope that weighs 24 pounds is hanging off a cliff. How much work is required to pull the rope up 2 ft?

(a) 42 ft-lb
(b) 6 ft-lb
(c) 54 ft-lb
(d) 18 ft-lb
(e) 48 ft-lb

rope weighs 3 lb/ft

\[W = \int f(y) \, dy\]
\[= \int_{0}^{2} (24 - 3y) \, dy\]
\[= 24y - \frac{3}{2} y^2 \bigg|_{0}^{2}\]
\[= 48 - 6\]
\[= 42 \text{ ft-lb}\]
13. Compute \( \int_{1}^{e} \frac{\ln x}{x^2} \, dx \).

\[
\begin{align*}
\text{(a)} & \quad \frac{2}{e} - 1 \\
\text{(b)} & \quad \frac{1}{2} \\
\text{(c)} & \quad \frac{8}{3e^3} + \frac{2}{3} \\
\text{(d)} & \quad \frac{8}{3e^3} - \frac{2}{3} \\
\text{(e)} & \quad \frac{2}{e} + 1
\end{align*}
\]

\[
\begin{align*}
uv - \int v \, du &= - \ln x + \int \frac{1}{x^2} \, dx \\
&= - \ln x - \frac{1}{x} \bigg|_{1}^{e} \\
&= \left( -\frac{1}{e} - \frac{1}{e} \right) - \left( 0 - 1 \right) \\
&= - \frac{2}{e} + 1
\end{align*}
\]

14. Find the volume of the solid found by rotating the region bounded by the curves \( y = \frac{1}{x^3} \), \( x = 0 \), \( y = 1 \), and \( y = 2 \) in the first quadrant about the \( y \)-axis.

\[
\begin{align*}
\text{(a)} & \quad \frac{7\pi}{24} \\
\text{(b)} & \quad \frac{\pi}{2} \\
\text{(c)} & \quad \pi \\
\text{(d)} & \quad \pi \ln 2 \\
\text{(e)} & \quad 2\pi \ln 2
\end{align*}
\]

\[
\text{Disk; } dy \\
r = \frac{1}{\sqrt{y}} \\
V = \int \pi r^2 \, dy \\
= \int_{1}^{2} \pi \left( \frac{1}{\sqrt{y}} \right)^2 \, dy \\
= \int_{1}^{2} \pi \left( \frac{1}{y} \right) \, dy = \pi \ln y \bigg|_{1}^{2} = \pi \ln 2
\]

15. Compute \( \int_{0}^{\pi/4} \tan^2 x \, dx \).

\[
\begin{align*}
\text{(a)} & \quad \frac{1}{6} \\
\text{(b)} & \quad 1 - \frac{\pi}{4} \\
\text{(c)} & \quad \frac{1}{6} - \frac{\pi}{4} \\
\text{(d)} & \quad \frac{1}{2} \\
\text{(e)} & \quad \text{None of these}
\end{align*}
\]

\[
\begin{align*}
\int_{0}^{\pi/4} \sec^2 x - 1 \, dx &= \left. \tan x - x \right|_{0}^{\pi/4} \\
&= 1 - \frac{\pi}{4}
\end{align*}
\]
Part 2: Work Out

**Directions:** Present your solutions in the space provided. **Show all your work** neatly and concisely and box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

16. (12 points) Consider the region bounded by the curves \( x = y^2 \) and \( x + 2y = 3 \).

(a) SET UP an integral to find the area of the region. **Do not evaluate your integral.**

\[
A = \int_{-3}^{1} (3-2y-y^2) \, dy
\]

(b) SET UP an integral to find the volume of the solid found by rotating this region about the line \( y = 6 \). **Do not evaluate your integral.**

Shell, \( dy \)

\[
V = \int_{-3}^{1} 2\pi r h \, dy = \int_{-3}^{1} 2\pi (6-y)(3-2y-y^2) \, dy
\]

(c) SET UP an integral to find the volume of the solid found by rotating this region about the line \( x = -3 \). **Do not evaluate your integral.**

Washer, \( dy \)

\[
V = \int_{-3}^{1} \pi [R^2 - r^2] \, dy = \int_{-3}^{1} \pi [(6-2y)^2 - (y^2 + 3)^2] \, dy
\]
17. (6 points) Compute \( \int \cos^5 x \sin^6 x \, dx \).

\[
\begin{align*}
\int \cos^5 x \sin^6 x \, dx &= \int \cos^4 x \sin^6 x \cos x \, dx \\
&= \int (1 - \sin^2 x)^2 \sin^6 x \cos x \, dx \\
&= \int (1 - u^2)^2 u^6 \, du \\
&= \int (u^6 - 2u^8 + u^{10}) \, du \\
&= \frac{1}{7} u^7 - \frac{2}{9} u^9 + \frac{1}{11} u^{11} + C \\
&= \frac{1}{7} \sin^7 x - \frac{2}{9} \sin^9 x + \frac{1}{11} \sin^{11} x + C
\end{align*}
\]

18. (8 points) A tank is in the shape of a trough with isosceles triangles as its ends. (See picture below). The trough is 14 m long, has a height of 7 m, and the width of the trough across the top is 4 m. The trough does not have a spout and is filled up to a height of 6 m. Set up an integral to find the work required to pump all the water out the top of the tank. Do not evaluate your integral. Note: The weight density of water is \( \rho g = 9800 \, \text{N/m}^3 \). Clearly indicate in the picture below where you are placing your axis and which direction is positive.

\[
\begin{align*}
\text{Area} &= lw = 14w = 14 \left( \frac{4}{7} y \right) = 8y \\
V_{\text{slice}} &= 8y \, dy \\
\text{distance} &= 7 - y \\
W &= \int_0^6 \rho g (7-y) 8y \, dy
\end{align*}
\]
19. (7 points) The base of a solid is the region bounded by the curves \( y = 3 - x^2 \) and the \( x \)-axis. Cross-sections perpendicular to the \( y \)-axis are triangles with height equal to twice the base. Find the volume of this solid.

\[
A = \frac{1}{2}bh = \frac{1}{2}b(2b) = b^2
\]

\[
b = \sqrt{3-y} - (-\sqrt{3-y}) = 2\sqrt{3-y}
\]

\[
A = b^2 = 4(3-y) = 12 - 4y
\]

\[
V = \int_0^3 (12 - 4y) \, dy = 12y - 2y^2 \bigg|_0^3 = 36 - 18 = \boxed{18}
\]

20. (7 points) Compute \( \int \arctan x \, dx \).

\[
\begin{array}{c|c}
\text{u} & \frac{dv}{dx} \\
\arctan x & 1 \\
\frac{1}{1+x^2} \, dx & x \\
\end{array}
\]

\[
u \, u - \int u \, du
\]

\[
x \arctan x - \int \frac{x}{1+x^2} \, dx \to u \text{-sub}
\]

\[
u = 1 + x^2
\]

\[
x \arctan x - \int \frac{1}{2} \cdot \frac{1}{u} \, du
\]

\[
\frac{1}{2} \, du = x \, dx
\]

\[
x \arctan x - \frac{1}{2} \ln |u| + C
\]

\[
x \arctan x - \frac{1}{2} \ln |1 + x^2| + C
\]

FOR INSTRUCTOR USE ONLY

<table>
<thead>
<tr>
<th>Question</th>
<th>Points Awarded</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-15</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>16</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>17</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>19</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>