MATH 152, Fall 2018
COMMON EXAM II - VERSION A

LAST NAME(print): ________________________ FIRST NAME(print): ________________________

INSTRUCTOR: ________________________

SECTION NUMBER: ______________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-16), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 17-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE HONOR CODE

"An Aggie does not lie, cheat or steal, or tolerate those who do."

By signing below, you are stating that on your honor as an Aggie, you have neither given nor received unauthorized aid on this exam.

Signature: ________________________
Part 1: Multiple Choice (4 points each)

1. Find the correct complete form for the partial fraction decomposition of \( f(x) = \frac{3}{x^2(x^2 - 4)(x^2 + 9)} \)?

   (a) \( \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 2} + \frac{D}{x + 2} + \frac{E}{x^2 + 9} \)

   (b) \( \frac{A}{x^2} + \frac{B}{x - 2} + \frac{C}{x + 2} + \frac{E}{x^2 + 9} \)

   (c) \( \frac{A}{x^3} + \frac{B}{x - 2} + \frac{C}{x + 2} + \frac{D}{x - \lambda} + \frac{E}{x + \lambda} \)

   (d) \( \frac{A}{x^2} + \frac{B}{x - 2} + \frac{C}{x + 2} + \frac{D}{x^2 + 9} \)

   (e) \( \frac{A}{x^2} + \frac{B}{x - 2} + \frac{C}{x + 2} + \frac{D}{(x + \lambda)^2} \)

2. For a series \( \sum_{n=1}^{\infty} a_n \), we are given that the sequence of partial sums is \( s_n = 2 + e^{1/n} \). Which of the following statements is true?

   (a) The series converges to 3.

   (b) The series converges to 2.

   (c) The series converges to 0.

   (d) The series converges to 1.

   (e) The series diverges.

   \[ \lim_{n \to \infty} S_n = \lim_{n \to \infty} 2 + e^{1/n} = 2 + e^0 = 3 \]

3. Which of the following is the correct trig substitution needed to evaluate the integral \( \int \sqrt{x^2 + 6x + 5} \, dx \)?

   (a) \( x + 3 = 2 \sin \theta \)

   (b) \( x + 3 = \sqrt{14} \sec \theta \)

   (c) \( x + 3 = \sqrt{14} \sin \theta \)

   (d) \( x + 3 = \sqrt{14} \tan \theta \)

   (e) \( x + 3 = 2 \sec \theta \)

   \( (x^2 + 6x + 9)^{1/2} + 5 - 9 \)

   \( (x + 3)^2 - 4 \)

   \( x + 3 = 2 \sec \theta \)

4. Compute \( \int_{2}^{3} x^3 e^{-x^4} \, dx \).

   (a) \( \frac{1}{4e^2} \)

   (b) \( \frac{1}{4e^{16}} \)

   (c) \( 4e^{16} \)

   (d) \( 4e^2 \)

   (e) The integral diverges.

   \[ \int_{2}^{3} x^3 e^{-x^4} \, dx = \lim_{t \to \infty} \left[ \left. \frac{u}{4} e^{u} \right|_{2}^{t} \right] \]

   \[ = \left. \frac{u}{4} e^{u} \right|_{2}^{t} + \frac{1}{4} e^{-16} \]

   \[ = -\frac{1}{4} e^{-t^4} + \frac{1}{4} e^{-16} \]

   \[ \lim_{t \to \infty} \left[ -\frac{1}{4} e^{-t^4} + \frac{1}{4} e^{-16} \right] = 0 + \frac{1}{4} e^{-16} \]
5. The recursive sequence defined below is both bounded and increasing. Determine the limit of the sequence.

\[ a_1 = 3; \quad a_{n+1} = \sqrt{6a_n} - 5 \]

(a) The sequence converges to 1.
(b) The sequence converges to 5.
(c) The sequence converges to 3.
(d) The sequence converges to 4.
(e) The sequence diverges.

\[ L = \sqrt{6L-5} \]

\[ L^2 = 6L - 5 \]

\[ L^2 - 6L + 5 = 0 \]
\[ (L - 5)(L - 1) = 0 \]
\[ L = 1, 5 \]
\[ a_1 = 3 \text{ and increasing, so} \quad L = 5 \]

6. Find the sum of the series \( \sum_{n=1}^{\infty} \frac{2^n}{3^{2n}} \).

(a) The series converges to \( \frac{2}{3} \).
(b) The series converges to \( \frac{9}{7} \).
(c) The series converges to \( 2 \).
(d) The series converges to \( \frac{2}{7} \).
(e) The series diverges.

\[ \sum = \frac{1st \ term}{1 - r} = \frac{\frac{2}{9}}{1 - \frac{1}{2}} = \frac{2}{7} \]

7. Use the Remainder Estimate for the Integral Test to determine the smallest number of terms needed to approximate the sum of the series \( \sum_{n=1}^{\infty} \frac{1}{(2n+4)^2} \) with error less than or equal to \( \frac{1}{100} \).

(a) 23 terms
(b) 3 terms
(c) 25 terms
(d) 10 terms
(e) 15 terms

\[ R_n = \int_{n+1}^{\infty} \frac{1}{(2x+4)^2} \, dx \leq \frac{1}{100} \]

\[ \left. \frac{-1}{2} \cdot \frac{1}{2x+4} \right|_{n}^{t} = \frac{-1}{2(2n+4)} + \frac{1}{2(2t+4)} \]

\[ \lim_{t \to \infty} \left[ \frac{-1}{2(2t+4)} + \frac{1}{2(2n+4)} \right] = \frac{1}{2(2n+4)} \]

\[ \frac{1}{2(2n+4)} \leq \frac{1}{100} \implies 2(2n+4) \geq 100 \]
\[ 2n + 4 \geq 50 \]
\[ 2n \geq 46 \]
\[ n \geq 23 \]
8. Evaluate \( \int_0^4 \frac{x + 2}{x^2 + 4} \, dx \):
\[
\begin{align*}
\int_0^4 \frac{x}{x^2+4} + \frac{2}{x^2+4} \, dx &= \left[ \frac{1}{2} \ln|x^2+4| + 2 \cdot \frac{1}{2} \arctan \left( \frac{x}{2} \right) \right]_0^4 \\
&= \left[ \frac{1}{2} \ln 20 + \arctan(2) \right] - \left[ \frac{1}{2} \ln 4 + \arctan(0) \right] \\
&= \frac{1}{2} \ln 20 - \frac{1}{2} \ln 4 + \arctan 2
\end{align*}
\]
(a) \( \ln 6 - \ln 2 \)
(b) \( \ln 20 - \ln 4 \)
(c) \( \frac{1}{2} (\ln 20 - \ln 4) + 2 \arctan(4) \)
(d) \( \ln 20 - \ln 4 + 2 \arctan(4) \)
(e) \( 3(\ln 20 - \ln 4) + \arctan 2 \)

9. Compute \( \int_0^5 \frac{1}{x-2} \, dx \):
\[
\begin{align*}
\text{Wt. at } x=2 & \rightarrow \int_0^2 \frac{1}{x-2} \, dx + \int_2^5 \frac{1}{x-2} \, dx \\
\int_0^2 \frac{1}{x-2} \, dx &= \ln|1-2| \bigg|_0^2 = \ln|1-2| - \ln 2 \\
\lim_{t \to 2^-} \left[ \ln|t-2| - \ln 2 \right] &= -\infty \quad \text{Diverges}
\end{align*}
\]
(a) \( \ln 3 - \ln 2 \)
(b) \( \ln 3 + \ln 2 \)
(c) \( \ln 3 \)
(d) \( \ln 2 \)
(e) The integral diverges.

10. Which of the following statements is true regarding the sequence \( a_n = \ln(n+1) - \ln(3n) \)?
\[
\begin{align*}
\text{(a) The sequence converges to 0.} \\
\text{(b) The sequence converges to } \ln(3). \\
\text{(c) The sequence converges to } \ln \left( \frac{1}{3} \right). \\
\text{(d) The sequence converges to 1.} \\
\text{(e) The sequence diverges.}
\end{align*}
\]

11. Which of the following series diverge by the Test for Divergence?
(I) \( \sum_{n=0}^\infty \frac{2n+1}{3n+4} \) \( \lim_{n \to \infty} \frac{2n+1}{3n+4} = \frac{2}{3} \neq 0 \quad \text{Diverges} \)
(II) \( \sum_{n=0}^\infty \left( \frac{1}{4} \right)^n \) \( \lim_{n \to \infty} \left( \frac{1}{4} \right)^n = 0 \quad \text{Converges} \)
(III) \( \sum_{n=1}^\infty \frac{n^2 + 1}{n+3} \) \( \lim_{n \to \infty} \frac{n^2+1}{n+3} = \infty \quad \text{Diverges} \)
(a) III only
(b) I only
(c) II and III only
(d) I and III only
(e) All three diverge by the Test for Divergence.
12. Evaluate \( \int_4^6 \frac{x^2}{x-3} \, dx \).

(a) \( 6 + 9 \ln 3 \)
(b) \( 4 + 9 \ln 3 \)
(c) \( 4 - 9 \ln 3 \)
(d) \( 4 \)
(e) \( 16 - 9 \ln 3 \)

\[
\begin{align*}
\int_4^6 \frac{x^2}{x-3} \, dx &= \left[ \frac{x}{x-3} + \frac{9}{x-3} \right]_4^6 \\
&= -\frac{3}{2}x^2 + 3x + 9 \ln |x-3| \bigg|_4^6 \\
&= (18 + 18 + 9 \ln 3) - (8 + 12 + 9 \ln 1) \\
&= 16 + 9 \ln 3
\end{align*}
\]

13. Which of the following integrals is obtained by making the correct trig substitution in evaluating \( \int \frac{x^2}{\sqrt{16 - x^2}} \, dx \)?

(a) \( \int \frac{4 \sec^2 \theta \tan \theta}{\tan \theta} \, d\theta \)
(b) \( \int \frac{4 \sin^2 \theta \cos \theta}{\cos \theta} \, d\theta \)
(c) \( \int \frac{4 \sin^2 \theta}{\cos \theta} \, d\theta \)
(d) \( \int \frac{16 \sin^2 \theta}{\cos \theta} \, d\theta \)
(e) \( \int \frac{16 \sin^2 \theta}{\cos \theta} \, d\theta \)

\[
\begin{align*}
\int \frac{x^2}{\sqrt{16 - x^2}} \, dx &= \int \frac{x^2}{\sqrt{16 - 16 \sin^2 \theta}} \, d\theta \\
&= \int \frac{16 \sin^2 \theta \cdot 4 \cos \theta}{4 \cos \theta} \, d\theta \\
&= \int 16 \sin^2 \theta \, d\theta
\end{align*}
\]

14. Which of the following statements is true regarding the sequences below?

\( a_n = \frac{(-1)^n n}{3n + 1} \) and \( b_n = \frac{(-1)^n \ln n}{n} \)

(a) Both sequences diverge.
(b) Both sequences converge.
(c) \( a_n \) converges; \( b_n \) diverges.
(d) \( a_n \) diverges; \( b_n \) converges.
(e) \( a_n \) diverges but the convergence/divergence of \( b_n \) cannot be determined.

\[
\begin{align*}
\lim_{n \to \infty} \frac{n}{3n+1} &= \frac{1}{3} \neq 0 \rightarrow a_n \text{ diverges} \\
\lim_{n \to \infty} \frac{\ln n}{n} &= 0 \rightarrow b_n \text{ converges}
\end{align*}
\]
15. Which of the following sequences is increasing and bounded?

(a) \( a_n = e^{-n} \)
(b) \( a_n = 1 - \frac{1}{n} \)
(c) \( a_n = \ln n \)
(d) \( a_n = \cos(n) \)
(e) None of these sequences are increasing and bounded.

16. Which of the following statements is true regarding the integral \( \int_2^\infty \frac{x + e^{-x}}{x^3} \, dx ? \)

(a) The integral diverges by comparison with \( \int_2^\infty \frac{1}{x} \, dx \).
(b) The integral converges by comparison with \( \int_2^\infty \frac{1}{x} \, dx \).
(c) The integral converges by comparison with \( \int_2^\infty \frac{1}{x^2} \, dx \).
(d) The integral diverges by comparison with \( \int_2^\infty \frac{1}{x^2} \, dx \).
(e) None of the above statements are true.

Part 2: Work Out

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (8 points) Consider the series \( \sum_{k=1}^\infty \left( e^{-(k+1)} - e^{-k} \right) \).

(a) Find a formula for \( s_n \), the \( n \)th partial sum of the series. Simplify your answer.

\[
\begin{align*}
\frac{1}{e} - \frac{1}{e^2} & + \frac{1}{e^3} - \frac{1}{e^2} + \cdots \frac{1}{e^{n+1}} - \frac{1}{e^n} \\
S_n &= -\frac{1}{e} + \frac{1}{e^{n+1}}
\end{align*}
\]

(b) Find the sum of the series or show it diverges.

\[
\lim_{n \to \infty} S_n = \lim_{n \to \infty} -\frac{1}{e} - \frac{1}{e^{n+1}} = -\frac{1}{e} - \frac{1}{e} = -\frac{2}{e}
\]
18. (10 points) Compute \( \int \frac{2x^2 - 5x + 9}{(x+2)(x-1)^2} \, dx \).

\[
\frac{2x^2 - 5x + 9}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}
\]

\[
2x^2 - 5x + 9 = A(x-1)^2 + B(x+2)(x-1) + C(x+2)
\]

\( x = 1 \):
\[ 6 = 3C \rightarrow C = 2 \]

\( x = -2 \):
\[ 8 + 10 + 9 = 9A \]
\[ 27 = 9A \rightarrow A = 3 \]

\( x = 0 \):
\[ 9 = A + B(2)(-1) + 2C \]
\[ 9 = 3 - 2B + 4 \]
\[ 2B = -2 \]
\[ B = -1 \]

\[
\int \left[ \frac{3}{x+2} - \frac{1}{x-1} + \frac{2}{(x-1)^2} \right] \, dx
\]

\[ = 3 \ln |x+2| - \ln |x-1| - \frac{2}{x-1} + C \]
19. (10 points) Compute \( \int \frac{1}{(4 + 9x^2)^{3/2}} \, dx \).

\[
\int \frac{\frac{2}{3} \sec^2 \theta}{(4 + 4 \tan^2 \theta)^{3/2}} \, d\theta \\
= \int \frac{\frac{2}{3} \sec^3 \theta}{(2 \sec \theta)^5} \, d\theta \\
= \int \frac{\frac{2}{3} \cdot \frac{1}{2^5} \cdot \frac{1}{\sec^3 \theta}}{\tan \theta} \, d\theta \\
= \frac{1}{48} \int \cos^3 \theta \, d\theta \\
= \frac{1}{48} \int (1 - \sin^2 \theta) \cos \theta \, d\theta \\
= \frac{1}{48} \int (1 - u^2) \, du \\
= \frac{1}{48} \left[ u - \frac{1}{3} u^3 \right] + C \\
= \frac{1}{48} \left[ \sin \theta - \frac{1}{3} \sin^3 \theta \right] + C \\
= \frac{1}{48} \left[ \frac{3x}{\sqrt{1 + x^2}} - \frac{1}{3} \left( \frac{3x}{\sqrt{1 + x^2}} \right)^3 \right] + C
\]
20. (8 points) Determine whether the series below converges or diverges.

\[
\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} \quad \lim_{n \to \infty} \frac{1}{n(\ln n)^3} = 0
\]

\[
f(x) = \frac{1}{x(\ln x)^3} \implies f \text{ is continuous, positive, and decreasing on } [2, \infty).
\]

**Integral Test:**

\[
\int_{2}^{\infty} \frac{1}{x(\ln x)^3} \, dx
\]

\[
\int_{2}^{t} \frac{1}{x(\ln x)^3} \, dx = \int_{2}^{t} \frac{1}{u^3} \, du = -\frac{1}{2u^2} \bigg|_{2}^{t} = -\frac{1}{2(\ln t)^2} + \frac{1}{2(\ln 2)^2}
\]

\[
\lim_{t \to \infty} \left[ -\frac{1}{2(\ln t)^2} + \frac{1}{2(\ln 2)^2} \right] = \frac{1}{2(\ln 2)^2}
\]

*Improper integral converges, so series also converges.*