DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 4 points.

4. In Part 2 (Problems 16-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________

DO NOT WRITE BELOW!

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PART I: Multiple Choice. 4 points each.

1. The improper integral \( \int_1^\infty \frac{\sin(x) + 3}{x^2} \, dx \)

(a) Diverges because \( \int_1^\infty \frac{3}{x^2} \, dx \) diverges and \( \frac{\sin(x) + 3}{x^2} \geq \frac{3}{x^2} \)

(b) Converges because \( \int_1^\infty \frac{2}{x^2} \, dx \) converges and \( \frac{\sin(x) + 3}{x^2} \leq \frac{2}{x^2} \)

(c) Diverges because \( \int_1^\infty \frac{4}{x^2} \, dx \) diverges and \( \frac{\sin(x) + 3}{x^2} \geq \frac{4}{x^2} \)

(d) Converges because \( \int_1^\infty \frac{1}{x^2} \, dx \) converges and \( \frac{\sin(x) + 3}{x^2} \leq \frac{1}{x^2} \)

(e) Converges because \( \int_1^\infty \frac{4}{x^2} \, dx \) converges and \( \frac{\sin(x) + 3}{x^2} \leq \frac{4}{x^2} \)

2. The curve \( x = 1 + \cos t, \; y = -2 + \sin t \)

(a) Is a circle centered at \((-1, 2)\) with radius 1, oriented clockwise.

(b) Is a circle centered at \((1, -2)\) with radius 1, oriented counterclockwise.

(c) Is a circle centered at \((-1, 2)\) with radius 1, oriented counterclockwise.

(d) Is a circle centered at \((1, -2)\) with radius 1, oriented clockwise.

(e) None of these

3. At which of the following points does \( x = 2t^3 + 3t^2 - 12t, \; y = t^2 - 4t \) have a vertical tangent?

(a) \((-7, -3)\) and \((20, 12)\) only

(b) \((4, -4)\) and \((13, 5)\) only

(c) \((-7, -3)\), and \((0, 0)\) only

(d) \((4, -4), (13, 5)\) and \((0, 0)\) only

(e) \((4, -4)\) only
4. \[ \int_{1}^{\infty} \frac{e^{1/x}}{x^2} \, dx = \]
   (a) 0
   (b) diverges
   (c) \( e \)
   (d) \( 1 - e \)
   (e) \( e - 1 \)

5. Which of the following integrals gives the arc length of the cardioid \( r = 1 + \sin \theta \)?
   (a) \[ \int_{0}^{\pi} \sqrt{2} \, d\theta \]
   (b) \[ \int_{0}^{2\pi} \sqrt{2} \, d\theta \]
   (c) \[ \int_{0}^{2\pi} \sqrt{2 + 2 \sin \theta} \, d\theta \]
   (d) \[ \int_{0}^{\pi} \sqrt{1 + 2 \sin \theta} \, d\theta \]
   (e) \[ \int_{0}^{2\pi} \sqrt{1 + 2 \sin \theta} \, d\theta \]

6. \[ \int_{3}^{4} \frac{x + 4}{x^2 - 2x} \, dx = \]
   (a) \( 2 \ln 4 - 3 \ln 2 - 2 \ln 3 \)
   (b) \( -2 \ln 4 + 3 \ln 2 + 2 \ln 3 \)
   (c) \( -2 \ln 4 + 3 \ln 2 \)
   (d) \( 2 \ln 4 + 3 \ln 2 + 2 \ln 3 \)
   (e) \( -2 \ln 4 + 3 \ln 2 - 2 \ln 3 \)
7. Which of the following is a polar equation of the circle \( x^2 + y^2 = 2x \)?

(a) \( r = 2 \)
(b) \( r = 2 \cos \theta \)
(c) \( r^2 = 2 \cos \theta \)
(d) \( r = 2 \sin \theta \)
(e) \( r^2 = 2 \sin \theta \)

8. \( \int_0^1 \frac{x^2 + 3x + 4}{x + 1} \, dx = \)

(a) \( \frac{3}{2} - 2 \ln 2 \)
(b) 4
(c) \( \frac{5}{2} + 2 \ln 2 \)
(d) \( \frac{5}{2} - 2 \ln 2 \)
(e) \( \frac{3}{2} + \ln 2 \)

9. Find the area of the part of the circle \( r = 4 \sin \theta \) that lies within the sector \( 0 \leq \theta \leq \frac{\pi}{3} \).

(a) \( \frac{4\pi}{3} - 1 \)
(b) \( \frac{8\pi}{3} - 2\sqrt{3} \)
(c) \( \frac{4\pi}{3} + \sqrt{3} \)
(d) \( \frac{4\pi}{3} - \sqrt{3} \)
(e) \( \frac{4\pi}{3} + 1 \)
10. Which of the following integrals gives the surface area obtained by rotating the curve \( x = e^{2t}, \ y = te^t, \ 0 \leq t \leq 1, \) about the \( y \)-axis?

(a) \( \int_0^1 2\pi e^{2t} \sqrt{4e^{4t} + (e^t + te^t)^2} \, dt \)

(b) \( \int_0^1 2\pi t e^t \sqrt{\frac{1}{4} e^{4t} + (e^t + te^t)^2} \, dt \)

(c) \( \int_0^1 2\pi e^{2t} \sqrt{e^{2t} + (e^t + te^t)^2} \, dt \)

(d) \( \int_0^1 2\pi e^{2t} \sqrt{\frac{1}{4} e^{4t} + (e^t + te^t)^2} \, dt \)

(e) \( \int_0^1 2\pi t e^t \sqrt{4e^{4t} + (e^t + te^t)^2} \, dt \)

11. Find the tangent line to the parametric curve \( x = \ln(t) + 4, \ y = 2t + t^2 \) at the point where \( t = 1. \)

(a) \( y = 4x - 13 \)

(b) \( y = 4x - 4 \)

(c) \( y = \frac{1}{4}x + 4 \)

(d) \( y = \frac{1}{4}x + \frac{13}{4} \)

(e) \( y = 4x - 16 \)

12. After applying the appropriate substitution, which of the following is equivalent to \( \int_0^{1/4} \sqrt{1 - 4x^2} \, dx? \)

(a) \( \int_0^{\pi/6} \cos \theta \, d\theta \)

(b) \( \int_0^{\pi/3} \cos \theta \, d\theta \)

(c) \( \frac{1}{2} \int_0^{\pi/3} \cos^2 \theta \, d\theta \)

(d) \( \frac{1}{2} \int_0^{\pi/6} \cos^2 \theta \, d\theta \)

(e) \( \int_0^{\pi/6} \cos^2 \theta \, d\theta \)
13. Which of the following integrals is/are improper?

I. $\int_{0}^{2} \frac{1}{2x - 1} \, dx$  
II. $\int_{1}^{\infty} \frac{1}{x^3} \, dx$  
III. $\int_{1}^{2} x \ln(2 - x) \, dx$

(a) II. and III. only
(b) I. and II. only
(c) II. only
(d) I. II. and III.
(e) I. and III. only

14. Which of the following is the appropriate trigonometric substitution for $\int \frac{dx}{\sqrt{x^2 + 6x + 13}}$?

(a) $x + 3 = \sqrt{13} \tan \theta$
(b) $x^2 + 6x = \sqrt{13} \sec \theta$
(c) $x + 3 = 4 \tan \theta$
(d) $x + 3 = 2 \sin \theta$
(e) $x + 3 = 2 \tan \theta$

15. The point $(-1, \sqrt{3})$ is equivalent to which of the following in polar coordinates?

(a) $\left(-2, \frac{5\pi}{6}\right)$
(b) $\left(-2, \frac{5\pi}{3}\right)$
(c) $\left(2, \frac{4\pi}{3}\right)$
(d) $\left(2, \frac{5\pi}{6}\right)$
(e) $\left(-2, \frac{2\pi}{3}\right)$
PART II: Work Out: Box your final answer!

16. (a) (5 pts) Find the general form of the partial fraction decomposition of
\[ f(x) = \frac{4x^3 + 5x + 10}{x^4 + 5x^2}, \]
and find the numerical values of the coefficients.

(b) (6 pts) Using the result of part a.), find
\[ \int \frac{4x^3 + 5x + 10}{x^4 + 5x^2} \, dx \]
17. (10 pts) Find $\int \frac{dx}{x^4\sqrt{x^2 - 9}}$
18. (7 pts) Find the length of the curve \( x = 2e^{2t} - t, \ y = 4e^t \) \( 0 \leq t \leq 2 \).

19. (6 pts) Consider the polar region \( R \) inside the circle \( r = 6 \sin \theta \) and outside the cardioid \( r = 2 + 2 \sin \theta \), as shown below. Set up but do not evaluate an integral that gives the area of \( R \).
20. (6 pts) Write the letter of the correct graph corresponding to the polar curve. No work needs to be shown.

_______ \( r = 2 \sin(3\theta) \)

_______ \( r = 2 \cos(4\theta) \)