DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-14), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 15-19), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE HONOR CODE

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

By signing below, you are stating that on your honor as an Aggie, you have neither given nor received unauthorized aid on this exam.

Signature: ________________________________
Part 1: Multiple Choice (4 points each)

1. The series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -2$ and diverges when $x = 5$. What can be said about the following series?

(I) $\sum_{n=0}^{\infty} c_n 2^n$  \hspace{1cm} (II) $\sum_{n=0}^{\infty} c_n (-6)^n$

(a) Both series diverge
(b) Both series converge
(c) I converges; II diverges
(d) I cannot be determined; II diverges
(e) Neither can be determined

2. Which of the following statements is true regarding the series $\sum_{n=0}^{\infty} \frac{n 2^n}{3^{2n}}$?

(a) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{2}{3}$ and the series converges by the Ratio Test.
(b) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{2}{3}$ and the series diverges by the Ratio Test.
(c) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{2}{9}$ and the series converges by the Ratio Test.
(d) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{2}{9}$ and the series diverges by the Ratio Test.
(e) $\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \infty$ and the series diverges by the Ratio Test.

3. Which of the following alternating series converge?

(I) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$  \hspace{1cm} (II) $\sum_{n=2}^{\infty} \frac{(-1)^n \ln n}{n}$

(a) Both series converge
(b) Both series diverge
(c) I only
(d) II only
(e) Not enough information
4. Determine the radius and interval of convergence for the power series \[ \sum_{n=0}^{\infty} \frac{(x - 1)^n}{5^n n!} \].

(a) \( R = 0, I = \{1\} \)
(b) \( R = 0, I = (-\infty, \infty) \)
(c) \( R = 5, I = (-4, 6) \)
(d) \( R = \frac{1}{5}, I = \left(\frac{4}{5}, \frac{6}{5}\right) \)
(e) \( R = \infty, I = (-\infty, \infty) \)

5. The Taylor series for a function \( f(x) \) centered at \( a = 2 \) is given by \[ \sum_{n=0}^{\infty} \frac{3n}{(n+1)!} (x - 2)^n \]. What is \( f^{(32)}(2) \), that is, the 32nd derivative of \( f(x) \) evaluated at \( a = 2 \).

(a) \( \frac{96}{33} \)
(b) \( \frac{96}{33!} \)
(c) \( \frac{96}{32! \cdot 33!} \)
(d) \( 96 \cdot 32! \)
(e) None of these

6. Which of the following statements is true regarding the series \[ \sum_{n=1}^{\infty} \frac{\sin n + 2}{n \sqrt{n}} \]?

(a) The series converges by the Comparison Test with the series \( \sum_{n=1}^{\infty} \frac{1}{n \sqrt{n}} \).
(b) The series converges by the Comparison Test with the series \( \sum_{n=1}^{\infty} \frac{3}{n \sqrt{n}} \).
(c) The series diverges by the Comparison Test with the series \( \sum_{n=1}^{\infty} \frac{3}{n \sqrt{n}} \).
(d) The series diverges by the Comparison Test with the series \( \sum_{n=1}^{\infty} \frac{1}{n \sqrt{n}} \).
(e) None of the above statements are true.
7. Use the Alternating Series Estimation Theorem to estimate the error in using \( s_5 \) (the sum of the first five terms) to approximate the sum of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \).

(a) \( \frac{1}{25} \)
(b) \( \frac{1}{5} \)
(c) \( \frac{1}{6} \)
(d) \( \frac{1}{36} \)
(e) \( \frac{1}{16} \)

8. Which of the following series converge?

(I) \( \sum_{n=2}^{\infty} \frac{n^2 - 3}{n^4 - n + 1} \)
(II) \( \sum_{n=1}^{\infty} \frac{\sqrt{n} + 1}{n + 7} \)

(a) I only
(b) II only
(c) Both series converge
(d) Both series diverge
(e) Not enough information

9. Find the sum of the series \( \sum_{n=0}^{\infty} \frac{3^{n+1}}{2^n n!} \).

(a) \( e^{3/2} \)
(b) \( 3e^{3/2} \)
(c) \( 3e^3 \)
(d) \( e^3 \)
(e) The series diverges

10. Which of the following is a power series centered at 0 for the function \( f(x) = \frac{x}{2 - x} \) ?

(a) \( \sum_{n=0}^{\infty} \frac{x^{n+1}}{2^n} \)
(b) \( \sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}} \)
(c) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{2^{n+1}} \)
(d) \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{2^n} \)
(e) \( \sum_{n=0}^{\infty} \frac{x^{n+1}}{2^{n+1}} \)
11. Find the Maclaurin series for the function \( f(x) = \arctan(x^2) \).

(a) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{2n+1} \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{2n+1} \]

(c) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{4n+1} \]

(d) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{2n+1} \]

(e) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+4}}{4n+4} \]

12. Find the radius of convergence for the Maclaurin series representation of \( f(x) = \frac{1}{1+9x^2} \).

(a) \( R = \frac{1}{3} \)
(b) \( R = 3 \)
(c) \( R = \infty \)
(d) \( R = 0 \)
(e) \( R = 1 \)

13. Find the third degree Taylor polynomial, \( T_3(x) \), centered at \( a = 1 \) for the function \( f(x) = \sqrt{x} \).

(a) \( T_3(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{8}(x-1)^3 \)

(b) \( T_3(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{3}{8}(x-1)^3 \)

(c) \( T_3(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 \)

(d) \( T_3(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{4}(x-1)^2 - \frac{3}{8}(x-1)^3 \)

(e) \( T_3(x) = 1 - \frac{1}{2}(x-1) + \frac{1}{8}(x-1)^2 - \frac{1}{16}(x-1)^3 \)
14. Consider two series with positive terms, \( \sum_{n=0}^{\infty} a_n \) and \( \sum_{n=0}^{\infty} b_n \). If \( \sum_{n=0}^{\infty} a_n \) diverges, which of the following is true?

(a) If \( b_n \leq a_n \) for all \( n \), then \( \sum_{n=0}^{\infty} b_n \) diverges.

(b) If \( b_n \leq a_n \) for all \( n \), then \( \sum_{n=0}^{\infty} b_n \) converges.

(c) If \( \lim_{n \to \infty} b_n = 0 \), then \( \sum_{n=0}^{\infty} b_n \) converges.

(d) If \( \lim_{n \to \infty} b_n = 0 \), then \( \sum_{n=0}^{\infty} b_n \) diverges.

(e) None of these statements are true.

Part 2: Work Out

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

15. (8 points) Determine whether the series below converges absolutely, converges but not absolutely, or diverges.

\[
\sum_{n=2}^{\infty} \frac{(-1)^n n}{n^2 + n + 1}
\]
16. (10 points) Find the radius and interval of convergence for the power series \[ \sum_{n=1}^{\infty} \frac{n(x + 3)^n}{4^n} \]. Be sure to check the endpoints for convergence.
17. (9 points) Evaluate the integral below as a power series using the known Maclaurin series for cosine.
\[ \int x^2 \cos(3x^2) \, dx \]

18. (9 points) Find the Taylor series for the function \( f(x) = \frac{1}{x} \) centered at \( a = 4 \). Express your answer in sigma notation.
19. (8 points) Find a power series centered at 0 for the function \( f(x) = \frac{1}{(2+3x)^2} \).