1. As \( n \to \infty \), \( \frac{\ln n}{\ln 2 + \ln n} \to 1 \).
2. The series \( \sum \frac{(-1)^n}{n^p} \) converges if and only if \( p > 0 \). Here’s why.
   - If \( p > 0 \), then \( |a_n| = \frac{1}{n^p} \to 0 \) and the series converges by the Alternating Series Test.
   - If \( p = 0 \), then \( |a_n| \to 1 \) whence \( \lim_{n \to \infty} a_n \neq 0 \) and the series diverges by the Test for Divergence.
   - If \( p < 0 \), then \( |a_n| = n^{-p} \to \infty \) so \( \lim_{n \to \infty} a_n \neq 0 \) and the series diverges by the Test for Divergence.
3. Let’s examine the statements one at a time.
   - I. is true. For assume that \( \sum a_n \) converges but \( \lim a_n \neq 0 \). Then \( \sum a_n \) diverges by the Test for Divergence, a contradiction. Therefore we must have \( \lim a_n = 0 \).
   - II. is false. For example, \( \lim \frac{1}{n} = 0 \), but \( \sum \frac{1}{n} = \infty \).
   - III. is false. So \( \sum_{n=1}^{\infty} \left( \frac{1}{3} \right) \left( \frac{4}{7} \right)^n \) is a geometric series, but with \( r = \frac{4}{7} \).
4. We have \( a_1 = s_2 = \frac{1}{4} - \frac{3}{2} = -\frac{5}{28} \) and
   \( \sum a_n = \lim_{n \to \infty} s_n = \lim_{n \to \infty} \frac{1 + 1}{3 - 2} = \frac{1}{3} \).
5. For \( \sum_{n=1}^{\infty} \frac{1}{n^2} \), we need \( R_n \leq \int_1^{n+1} \frac{dx}{x^2} = \frac{1}{n} < \frac{1}{10} \). The smallest positive integer for which this is true is 5 since \( \frac{1}{5} = \frac{1}{7} = \frac{1}{10} \) but \( \frac{1}{4} = \frac{1}{6} > \frac{1}{10} \). So add 5 terms.
6. We have \( \frac{\cos(2\pi n)}{\sqrt[n]{n}} \geq \sum \frac{2}{\sqrt{n}} \), a multiple of a divergent \( p \)-series \( (p = \frac{1}{2} \leq 1) \). So series diverges by Comparison Test.
7. We have \( \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^n = \sum_{n=0}^{\infty} \left( -\frac{2}{25} \right) \left( -\frac{3}{25} \right)^k = \frac{2}{25} \sum_{k=0}^{\infty} \left( -\frac{3}{5} \right)^k \).
   - 8. Note \( a_n = \frac{(-1)^n(1 + \frac{1}{n})}{\frac{1}{2} - 2} \). So we have lim sup \( a_n = \frac{1}{2} \) whereas
   \( \lim \inf a_n = -\frac{1}{2} \). So \( \lim a_n \) DNE. The sequence \( \{a_n\} \) diverges by the Test for Divergence.
9. Completing squares yields \( 2(x - 3)^2 + (y + 4)^2 = 44 \). The center is \( (3, -4) \).
10. The sequence \( \{ -\frac{1}{n} \} \) lies in the interval \([0, 1] \), so it is bounded. Moreover, as \( n \) increases, less is being subtracted from 1, so the terms of the sequence increase.
11. For \( \sum a_n = \sum \frac{3^n}{(n+1)!} \), we have \( \frac{a_{n+1}}{a_n} = \frac{3^{n+1} \cdot \frac{(n+1)!}{3^n}}{1} = \frac{3}{3} \to 0 < 1 \). So \( \sum a_n \) converges absolutely by the Ratio Test and hence \( \sum a_n \) converges.
12. For \( \sum_{n=0}^{\infty} \frac{x + 2)^n}{x - (2)^n} < 1 \) to get \(-3 < x < -1 \) for which the sum of this geometric series is \( \frac{1}{x^2} \).
13. Since the sequence \( a_n = \frac{4}{a_{n+1}} = \frac{10 - \frac{16}{dx}}{a_{n+1}} \to L \) so \( L = 10 - \frac{1}{16} \) or \( L^2 - L + 16 = 0 \). Thus \( L = 2 \) or \( L = 8 \). Since the sequence is bounded below by 4, we must have \( L = 8 \).
14. Now \( r = \frac{2/5}{1-(8/5)\cos 0} \), so \( e = \frac{2}{3} \), signifying a hyperbola.
15. Since \( \int_1^{\infty} e^{-x^2} dx = \frac{1}{2} \), the series \( \sum_{n=1}^{\infty} e^{-n^2} \) converges by the Integral Test.
16. Via the Alternating Series Estimation Theorem, \( |R_4| \leq |a_5| \).
17. Considering \( \sum a_n = \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n} \), let \( f(x) = \ln x \). Then
   \( f'(x) = \frac{1}{x} > 0 \) for \( x > 1 \). Also,
   \( \lim_{x \to 1} \frac{\ln x}{x} = \lim_{x \to 1} \frac{1}{1} = 1 \).
   - Thus ultimately (after the first 3 terms), \( b_n = |a_n| \) converges by the Alternating Series Test.
   - Now let’s consider \( \sum |a_n| \).
   - Since \( \int_1^{\infty} \frac{\ln x}{x^2} dx = \lim_{x \to 1} \frac{\ln x}{x} = 1 \), we conclude that \( \sum |a_n| \) diverges via the Integral Test.
   - Therefore, the series \( \sum a_n \) is conditionally convergent.
18. The Ratio Test gives
   \( \frac{|a_{n+1}|}{|a_n|} = \left| \frac{\frac{1}{n+1}}{\frac{1}{n}} \right| = \left| \frac{n}{n+1} \right| = \frac{n}{n+1} \to 1 > 1 \).
   [Or use Root Test: \( \sqrt[n]{|a_n|} = \frac{5}{3n^{n/3}} \to \frac{5}{3} > 1 \).]
19. The formula \( x_n = (-1)^n \frac{n^2}{1 + n} \), \( n \geq 1 \), generates the sequence \{ \( \frac{1}{2}, -\frac{4}{5}, \frac{9}{8}, -\frac{16}{11}, \ldots \) \}.
20. For \( \sum_{n=1}^{\infty} \left( \frac{\cos \left( \frac{1}{n+1} \right)}{3} - \cos \left( \frac{1}{n+1} \right) \right) \), we have the following.
   (a) \( s_n = \cos \left( \frac{1}{n} \right) - \cos \left( \frac{1}{n+1} \right) \)
   (b) The sum of the series is \( \lim_{n \to \infty} s_n = \cos \left( \frac{1}{n} \right) - 1 \).
21. We examine three series.
   (a) For \( \sum_{n=1}^{\infty} \frac{\cos \left( \frac{1}{n+1} \right)}{3} \), we let \( \frac{1}{n} = \sum \frac{1}{n+1} \). Then \( \lim_{n \to \infty} \frac{\cos \left( \frac{1}{n+1} \right)}{3} = 2 > 0 \).
   (b) \( \sum_{n=1}^{\infty} \frac{1}{2^n} \) is bounded and \( \sum_{n=1}^{\infty} \frac{1}{2^n} = 2 > 0 \), a multiple of a convergent \( p \)-series \( (p = 2 > 1) \). Hence the series converges by the Limit Comparison Test.
   (c) For \( \frac{1}{\arctan(n)} \), we have \( a_n = \frac{1}{\frac{1}{\pi}} \neq 0 \). The series diverges by the Test for Divergence.