MATH 152 Fall 2019
COMMON EXAM I - VERSION B

LAST NAME: Key to post FIRST NAME: ____________________________
INSTRUCTOR: ____________________________
SECTION NUMBER: ____________
UIN: ____________________________

DIRECTIONS:

1. The use of a calculator, laptop or cell phone is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-15), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore for your own records, also record your choices on your exam! Each problem is worth 4 points.

4. In Part 2 (Problems 16-19), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to write your name, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________________
PART I: Multiple Choice. 4 points each.

1. Evaluate \( \int_0^{\pi/8} \sin^2(2x) \, dx \).
   
   (a) \( \frac{\pi}{16} + \frac{1}{8} \)
   
   (b) \( \frac{\pi}{16} - \frac{\sqrt{2}}{8} \)
   
   (c) \( \frac{\pi}{16} + \frac{\sqrt{2}}{8} \)
   
   (d) \( \frac{\pi}{16} - \frac{1}{4} \)
   
   (e) \( \frac{\pi}{16} - \frac{1}{8} \)

2. Which of the following is the correct set up that gives the volume of the solid obtained by rotating the region bounded by \( y = x^2 \) and \( y = 1 \) about the x-axis?
   
   (a) \( 4\pi \int_0^1 (1-y)\sqrt{y} \, dy \)
   
   (b) \( \pi \int_{-1}^1 x^4 \, dx \)
   
   (c) \( 4\pi \int_0^1 y\sqrt{y} \, dy \)
   
   (d) \( \pi \int_{-1}^1 (1-x^2)^2 \, dx \)
   
   (e) \( 2\pi \int_0^1 y\sqrt{y} \, dy \)

   Note: if we use washers, \( R = 1, r = x^2 \)

   \[ V = \int_{-1}^1 \pi (1 - x^4) \, dx, \] which is not a distractor

3. Evaluate \( \int_0^{\pi/4} \tan^4 x \, dx \).
   
   (a) \( \frac{5}{22} \)
   
   (b) \( \frac{2}{35} \)
   
   (c) \( \frac{11}{30} \)
   
   (d) \( \frac{4}{35} \)
   
   (e) \( \frac{12}{35} \)

   \[ \int_0^{\pi/4} \tan^4 x \, dx = \int_0^{\pi/4} \tan^2 x \sec^2 x \, dx \]

   \[ = \int_0^{\pi/4} (u^2 + u^4) \, du \]

   \[ = (u^3/3 + u^5/5) \bigg|_0^{1/2} = \frac{12}{35} \]
4. Which of the following integrals gives the area of the region bounded by the curves \( x = y^2 \) and \( x = 6 - y^2 \):

(a) \( \int_{-3}^{2} (6 - y - y^2) \, dy \)

(b) \( \int_{4}^{0} (6 - x - \sqrt{x}) \, dx \)

(c) \( \int_{-3}^{2} (y^2 - 6 + y) \, dy \)

(d) \( \int_{4}^{0} (\sqrt{x} - 6 + x) \, dx \)

(e) None of the above

5. Evaluate \( \int_{0}^{1} x e^{1-x^2} \, dx. \)

(a) \( \frac{1}{2} (e - 1) \)

(b) \( \frac{1}{2} (1 - e) \)

(c) \( e - 1 \)

(d) \( 1 - e \)

(e) None of the above

6. Find the volume of the solid obtained by rotating the region bounded by \( y = \sqrt{x} \) and \( y = x \) about the \( x \)-axis.

(a) \( \frac{\pi}{6} \)

(b) \( \frac{11\pi}{6} \)

(c) \( \frac{4\pi}{15} \)

(d) \( \frac{2\pi}{15} \)

(e) None of the above
7. A rope that is 80 feet long and weighs 60 pounds hangs over a building 500 feet tall. How much work is done in pulling only the first 4 feet of rope to the top of the building?

(a) 246 foot pounds
(b) 6 foot pounds
(c) $\frac{688}{3}$ foot pounds
(d) $\frac{116}{3}$ foot pounds
(e) 234 foot pounds

$$W = \int_0^4 \left(60 - \frac{3}{8} x^2\right) dx$$

$$= \left[60 x - \frac{3}{8} x^3\right]_0^4$$

$$= 240 - \frac{3}{8} (16), \text{ or } 234 \text{ ft-lbs}$$

8. A spring has a natural length of 1 m. If a 40-N force is required to keep the spring stretched to a length of 3 m, how much work is done in stretching the spring from 1 m to 5 m?

(a) 240 J
(b) 160 J
(c) $\frac{320}{3}$ J
(d) 180 J
(e) None of the above

$$W = \int_0^4 20 x \, dx = 10 x^2 \bigg|_0^4$$

$$= 160 \text{ J}$$

9. Which of the following is the correct result after making the appropriate $u$-substitution to evaluate $\int_{\sqrt{2}/2}^1 \frac{x}{\sqrt{2x + 1}} \, dx$?

(a) $\frac{1}{2} \int_{3/2}^1 \frac{u - 1}{\sqrt{u}} \, du$
(b) $\frac{1}{2} \int_3^9 \frac{u - 1}{\sqrt{u}} \, du$
(c) $\int_4^9 \frac{u - 1}{\sqrt{u}} \, du$
(d) $\frac{1}{4} \int_{3/2}^1 \frac{u - 1}{\sqrt{u}} \, du$
(e) $\frac{1}{4} \int_4^9 \frac{u - 1}{\sqrt{u}} \, du$
10. Which of the following is the correct set up for the area of the region bounded by \( y = x^4 \) and \( y = 2 - |x| \), as shown in the figure below?

\[ A = \int_{-1}^{0} (2 + x - x^4) \, dx + \int_{1}^{0} (2 - x - x^4) \, dx \]

(a) \( \int_{-1}^{1} (2 + x - x^4) \, dx + \int_{0}^{1} (2 + x - x^4) \, dx \)

(b) \( \int_{-2}^{0} (2 + x - x^4) \, dx + \int_{0}^{2} (2 - x - x^4) \, dx \)

(c) \( \int_{-2}^{0} (2 - x - x^4) \, dx + \int_{0}^{2} (2 + x - x^4) \, dx \)

(d) \( \int_{-1}^{0} (2 + x - x^4) \, dx + \int_{0}^{1} (2 - x - x^4) \, dx \)

(e) None of the above

11. Find the volume of the solid obtained by rotating the region bounded by \( y = e^x \), \( y = 0 \), \( x = 0 \) and \( x = 1 \) about the \( x \)-axis.

\[ V = \pi \int_{0}^{1} (e^x)^2 \, dx \]

(a) \( \frac{\pi}{2}(e^2 - 1) \)

(b) \( 2\pi(e^2 - 1) \)

(c) \( \frac{\pi e^2}{2} \)

(d) \( 2\pi e^2 \)

(e) None of the above

12. Evaluate \( \int_{0}^{\pi/2} \sin^3(x) \cos^3(x) \, dx \).

(a) \( \frac{1}{12} \)

(b) \( \frac{1}{12} \)

(c) 0

(d) 1

(e) \( \frac{1}{4} \)

Since both powers are odd, I can factor out a \( \sin x \) or \( \cos x \).

\[ u = \sin x \quad \Rightarrow \quad \frac{du}{dx} = \cos x \quad \Rightarrow \quad dx = \frac{du}{\cos x} \]

\[ V = \pi \int_{0}^{\pi/2} \sin^3(x) \cos^2(x) \, dx \]

\[ = \pi \int_{0}^{\pi/2} \sin x(1 - \cos^2(x)) \cos x \, dx \]

\[ = \pi \int_{0}^{\pi/2} (u^3 - u) \, du \]

\[ = \left[ \frac{u^4}{4} - \frac{u^2}{2} \right]_{0}^{1} \]

\[ = \frac{1}{12} \]
13. Find the area of the region bounded by $y = 5x - x^2$ and $y = x$.

(a) $\frac{16}{3}$
(b) $\frac{32}{3}$
(c) $\frac{25}{3}$
(d) $\frac{40}{3}$
(e) None of the above

\[ 5x - x^2 = x \rightarrow 0 = x^2 - 4x \]
\[ x = 0, x = 4 \]

Curves intersect at $(0, 0)$ and $(4, 4)$

\[ A = \int_0^4 (5x - x^2 - x) \, dx \]
\[ = \int_0^4 (4x - x^2) \, dx \]
\[ = \left[ 2x^2 - \frac{x^3}{3} \right]_0^4 \quad \text{or} \quad \frac{32}{3} \]

14. Evaluate $\int_0^{\pi/3} x \cos x \, dx$.

(a) $\frac{1}{2} - \frac{\pi \sqrt{3}}{6}$
(b) $\frac{3}{2} + \frac{\pi \sqrt{3}}{6}$
(c) $-\frac{1}{2} + \frac{\pi \sqrt{3}}{6}$
(d) $\frac{\sqrt{3}}{2} + \frac{\pi}{6} - 1$
(e) $\frac{\sqrt{3}}{2} + \frac{\pi}{6} + 1$

\[ \text{parts: } u = x, \ du = \cos x \, dx \]
\[ \text{du} = dx, \ \nu = \sin x \]
\[ \int_0^{\pi/3} x \cos x \, dx = \left[ x \sin x \right]_0^{\pi/3} - \int_0^{\pi/3} \sin x \, dx \]
\[ = \left. \frac{\pi}{3} \left( \frac{\sqrt{3}}{2} \right) + (\cos x) \right|_0^{\pi/3} \]
\[ = \frac{\pi \sqrt{3}}{6} + \frac{1}{2} - 1 \quad \text{or} \quad \frac{\sqrt{3} \pi - 1}{2} \]

15. Evaluate $\int_1^4 \ln (\sqrt{x}) \, dx$. (Hint: Use logarithm properties to rewrite the integrand before integrating).

(a) $2 \ln 4 + \frac{5}{2}$
(b) $\ln 4 - \frac{3}{2}$
(c) $\ln 4 + \frac{3}{2}$
(d) $2 \ln 4 - \frac{3}{2}$
(e) $2 \ln 4 - 3$

\[ \ln \sqrt{x} = \ln x^{\frac{1}{2}} = \frac{1}{2} \ln x \]
\[ = \frac{1}{2} \int_1^4 \ln x \, dx \quad \text{parts: } u = \ln x, \ du = dx \]
\[ \text{du} = \frac{1}{x} \, dx, \ \nu = x \]
\[ \frac{1}{2} \left( \ln x \right|_1^4 + \int_1^4 \frac{x}{x} \, dx \right) \]
\[ = \frac{1}{2} \left( 4 \ln 4 - 4 - (0 - 1) \right) \]
\[ = 2 \ln 4 - \frac{3}{2} \]
PART II: Work Out: Box your final answer!

16. Consider the region bounded by \( y = x^3 \) and \( y = \sqrt{x} \). Set up but do not evaluate an integral that gives the volume of the solid obtained by revolving the specified region about the given line. DO NOT INTEGRATE.

(a) (5 pts) About the line \( x = -3 \) using the method of cylindrical shells.

\[
r = x - (-3) = x + 3 \\
h = \sqrt{x} - x^3 \\
V = \int_{0}^{1} 2\pi (x+3)(\sqrt{x} - x^3) \, dx
\]

(b) (5 pts) About the line \( y = 5 \) using the method of washers.

\[
r = 5 - x^3 \\
r = 5 - \sqrt{x} \\
V = \int_{0}^{1} \pi \left( (5-x^3)^2 - (5-\sqrt{x})^2 \right) \, dx
\]

17. (10 pts) Consider the tank shown below, which is full of water of weight density \( \rho g = 62.5 \) pounds per cubic foot. The end of the tank is in the shape of a semi-circle with radius 3 feet. Set up the integral that gives the work required to pump the water out of a 1 foot high spout at the top of the tank. Indicate on the picture where you are placing the axis and which direction is positive. DO NOT INTEGRATE.

By symmetry, the width of the slice is \( 2x \), where \( x^2 + y^2 = 9 \)

\[
x = \sqrt{9 - y^2}
\]

This slice of water has length 10 ft, height \( dy \) feet, and width which is shown in the figure below.

This slice of water weighs

\[
F = \rho g V, \text{ where } V = (2x)(10)(dy)
\]

\[
= 20 \sqrt{9-y^2} \, dy
\]

and must move a distance of

\[
d = 3-y+1 = 4-y
\]

to pass through the spout, since \( W = Fd \)

\[
W = \int_{0}^{3} (62.5)20\sqrt{9-y^2}(4-y) \, dy
\]
18. (10 pts) Consider the solid $S$ whose base is the region bounded by $x = y^2$ and $x = 9$, as shown in the figure below. Cross sections perpendicular to the $y$-axis are squares. What is the volume of $S$? \text{DO NOT SIMPLIFY.}

\[ V = 2 \int_0^3 S^2 \, dy \]
\[ = 2 \int_0^3 (9 - y^2)^2 \, dy \]
\[ = 2 \int_0^3 (81 - 18y^2 + y^4) \, dy \]
\[ = 2 \left[ 81y - 6y^3 + \frac{y^5}{5} \right]_0^3 \]
\[ \Rightarrow \left[ 2 \left( 81 \cdot 3 - 6 \cdot 3^3 + \frac{3^5}{5} \right) \right] \quad \text{or} \quad \frac{1296}{5} \]

19. (10 pts) Consider the region bounded by $y = \frac{1}{x}$, $y = x$ and $y = \frac{1}{16}x$, $x \geq 0$, as shown in the figure below. Set up but do not evaluate an integral (or sum of integrals) that gives the area of this region. \text{DO NOT INTEGRATE.}

\[
 A = \int_0^1 (x - \frac{1}{16}x) \, dx + \int_1^4 \left( \frac{1}{x} - \frac{1}{16}x \right) \, dx 
\]