DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-16), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!
4. In Part 2 (Problems 17-20), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
5. Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ____________________________
PART I: Multiple Choice. 4 points each

1. After an appropriate substitution, the integral \( \int_{-1}^{4} \frac{x}{(x+7)^5} \, dx \) is equivalent to which of the following?

(a) \( \int_{-1}^{1} (u^2 - 7u^{-3}) \, du \)
(b) \( \int_{6}^{11} (u^{-2} - 7u^{-3}) \, du \)
(c) None of these
(d) \( \int_{6}^{11} (7u^{-3} - u^{-2}) \, du \)
(e) \( \int_{-1}^{4} xu^{-3} \, du \)

The worked out solutions to the multiple choice question can be found on version A.

2. A spring has a natural length of 2 m. If a force of 12N is required to hold the spring to a length of 4m, find the work done to stretch the spring from 3m to 6m.

(a) 40.5 J
(b) 22.5 J
(c) 81 J
(d) 45 J
(e) None of these.

3. Which of the following represents the area between the curves \( y = 2x \), \( x + y = 6 \), and \( y = 0 \)?

(a) None of these.
(b) \( \int_{0}^{4} \frac{3y}{2} - 6 \, dy \)
(c) \( \int_{0}^{2} 6 - 3x \, dx \)
(d) \( \int_{0}^{2} 3x - 6 \, dx \)
(e) \( \int_{0}^{4} 6 - \frac{3y}{2} \, dy \)
4. Consider the region bounded by the curves $y = x^3$, $y = 8$, and the $y$-axis. Which of the following represents the volume of this region being rotated about the $x$-axis?

(a) $\int_0^8 \pi y^{2/3} \, dy$

(b) $\int_0^2 \pi (64 - x^6) \, dx$

(c) $\int_0^2 \pi (8 - x^3) \, dx$

(d) None of these.

(e) $\int_0^2 \pi (8 - x^3) \, dx$

5. Compute $\int_0^{\pi/2} \cos^2(\theta) \sin^2(\theta) \, d\theta$

(a) $\frac{4}{5}$

(b) $\frac{8}{15}$

(c) $\frac{2}{15}$

(d) None of these.

(e) $\frac{2}{5}$

6. Which of the following represents the area that is bounded between the curves $y = e^x$ and $y = e^{-x}$ on the interval $-1 \leq x \leq 2$?

(a) $\int_{-1}^{0} e^x \, dx + \int_{0}^{2} e^{-x} \, dx$

(b) $\int_{-1}^{0} e^{-x} - e^x \, dx + \int_{0}^{2} e^x - e^{-x} \, dx$

(c) $2 \int_{0}^{2} e^x - e^{-x} \, dx$

(d) $\int_{0}^{2} e^x - e^{-x} \, dx + \int_{-1}^{0} e^{-x} - e^x \, dx$

(e) None of these.
7. Compute \( \int_{0}^{1} xe^{(x^2+1)} \, dx \)

(a) \( \frac{1}{2} e^2 \)
(b) \( 2(e^2 - e) \)
(c) \( \frac{1}{2} (e - 1) \)
(d) \( \frac{1}{2} (e^2 - e) \)
(e) \( 2(e - 1) \)

8. Find the volume of the solid obtained by rotating the region bounded by the curves \( y = \frac{4}{x} \), \( y = x \), and \( x = 1 \) about the \( y \)-axis?

(a) \( \frac{16\pi}{6} \)
(b) \( \frac{5\pi}{3} \)
(c) \( \frac{4\pi}{3} \)
(d) \( \frac{10\pi}{3} \)
(e) \( \frac{5\pi}{6} \)

9. Compute \( \int_{0}^{\pi} x^2 \sin(x) \, dx \)

(a) \( \pi^2 + 4 \)
(b) None of these
(c) \( \pi^2 - 4 \)
(d) \( -\pi^2 + 4 \)
(e) \( -\pi^2 - 4 \)
10. A cable, 20 feet long and weighing 6 pounds per foot, is hanging off the side of a 30 foot tall building. At the bottom of the cable is a bucket of rocks weighing 100 pounds. How much work is required to pull 10 feet of the cable to the top of the building?

(a) 1900 ft-lbs
(b) 300 ft-lbs
(c) 3200 ft-lbs
(d) 900 ft-lbs
(e) 1300 ft-lbs

11. Consider the region bounded by the curves $y = \sqrt{x}$, $y = 0$ and $x = 4$. Which of the following represents the volume of this region being rotated about $x = -3$.

(a) $\int_{0}^{2} \pi (y^4 + 20y^2 - 40) \, dy$
(b) $\int_{0}^{2} \pi (58 - y^4 + 6y^2) \, dy$
(c) None of these.
(d) $\int_{0}^{2} \pi (y^4 - 20y^2 + 100) \, dy$
(e) $\int_{0}^{2} \pi (40 - y^4 - 6y^2) \, dy$

12. Compute $\int 8 \cos^2(\theta) \, d\theta$

(a) $4\theta + 2 \sin(2\theta) + C$
(b) None of these.
(c) $8\theta + 4 \sin(2\theta) + C$
(d) $4\theta - 2 \sin(2\theta) + C$
(e) $\frac{8}{3} \cos^3 \theta + C$
13. Consider the region bounded by the curves \( x = y^2 - 2y \) and the \( y \)-axis. Which of the following represents the volume of this region being rotated about \( y = 4 \).

(a) \( \int_{0}^{2} 2\pi(y - 4)(2y - y^2)dy \)

(b) \( \int_{0}^{2} 2\pi y(2y - y^2)dy \)

(c) \( \int_{0}^{2} 2\pi(4 - y)(y^2 - 2y)dy \)

(d) \( \int_{0}^{2} 2\pi y(y^2 - 2y)dy \)

(e) \( \int_{0}^{2} 2\pi(4 - y)(2y - y^2)dy \)

14. Compute \( \int_{1}^{e} \frac{(1 + \ln x) \ln x}{x} \, dx \)

(a) \( \frac{5}{6} \)

(b) \( \frac{1}{6} \)

(c) \( \frac{e^3}{3} + \frac{e^2}{2} - \frac{5}{6} \)

(d) \( \frac{3}{2} \)

(e) \( \frac{e^3}{3} - \frac{e^2}{2} + \frac{1}{6} \)

15. Compute \( \int x^{-3} \ln(5x) \, dx \)

(a) \( -\frac{x^{-4} \ln(5x)}{4} - \frac{x^{-4}}{16} + C \)

(b) \( -\frac{x^{-2} \ln(5x)}{2} - \frac{x^{-2}}{20} + C \)

(c) None of these.

(d) \( -\frac{x^{-4} \ln(5x)}{4} - \frac{x^{-4}}{80} + C \)

(e) \( -\frac{x^{-2} \ln(5x)}{2} - \frac{x^{-2}}{4} + C \)
16. Compute \( \int \tan^3(x) \sec(x) \, dx \)

(a) \( \frac{1}{3} \tan^3(x) - \tan(x) + C \)
(b) None of these.
(c) \( \frac{1}{3} \tan^3(x) + \tan(x) + C \)
(d) \( \frac{1}{3} \sec^3(x) - \sec(x) + C \)
(e) \( \frac{1}{3} \sec^3(x) + \sec(x) + C \)

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**PART II WORK OUT**

**Directions:** Present your solutions in the space provided. *Show all your work neatly and concisely and box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (8 points) Compute \( \int \sin(x)e^{2x} \, dx \)

\[
\int \sin(x)e^{2x} \, dx = -\cos(x)e^{2x} + 2\sin(x)e^{2x} - \frac{1}{2} \int \sin(x)e^{2x} \, dx
\]

\[
5 \int \sin(x)e^{2x} \, dx = -\cos(x)e^{2x} + 2\sin(x)e^{2x}
\]

\[
\int \sin(x)e^{2x} \, dx = \frac{1}{5} \left[ -\cos(x)e^{2x} + 2\sin(x)e^{2x} \right] + C
\]

---

\[
\int \sin(x)e^{2x} \, dx = \frac{1}{2} \sin(x)e^{2x} - \frac{1}{2} \cos(x)e^{2x} - \frac{1}{2} \int \sin(x)e^{2x} \, dx
\]

\[
\frac{5}{4} \int \sin(x)e^{2x} \, dx = \frac{1}{2} \sin(x)e^{2x} - \frac{1}{4} \cos(x)e^{2x}
\]

\[
\int \sin(x)e^{2x} \, dx = \frac{4}{5} \left[ \frac{1}{2} \sin(x)e^{2x} - \frac{1}{4} \cos(x)e^{2x} \right] + C
\]

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\[
\frac{1}{5} \frac{1}{2} \frac{1}{4} \frac{1}{2} \frac{1}{2}
\]

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18. (9 points) Compute \( \int \sec^4(x) \tan^4(x) \, dx \):

\[
\int \sec^4(x) \tan^4(x) \, dx = \int \sec^2(x) \tan^4(x) \, dx \\
= \int (\tan^2(x) + 1) \tan^4(x) \, dx \\
= \int \tan^6(x) \, dx \\
= \frac{\tan^7(x)}{7} + \frac{\tan^5(x)}{5} + C \\
= \frac{1}{7} \tan^7(x) + \frac{1}{5} \tan^5(x) + C
\]

19. (9 points) Find the volume of the solid whose base is the ellipse \( x^2 + 4y^2 = 4 \) and whose cross-sections perpendicular to the \( y \)-axis are squares. Evaluate your integral.

Base: \( 2x \)

\[
A = b^2 = (2x)^2 = 4x^2 \\
= 4(y-4y^2) \\
= 4y - 16y^2 = 16(1-y^2)
\]

\[
V = 16 \int_{-1}^{1} (1-y^2) \, dy = 2 \cdot 16 \int_{0}^{1} (1-y^2) \, dy = 2 \cdot 16 \left[ y - \frac{y^3}{3} \right]_{0}^{1} \\
= 32 \left( 1 - \frac{1}{3} \right) = 32 \left( \frac{2}{3} \right) = \frac{64}{3}
\]
20. (10 points) A hemispherical tank has the shape shown below. The tank has a radius of 7 meters with a 4 meter spout at the top of the tank. The tank is filled with water to a depth of 2 meters. The weight density of water is \( \rho g = 9800 \text{N/m}^3 \).

Set up an integral that will compute the work required to pump all the water out of the spout. **Do not evaluate!**

Clearly indicate on the picture where you are placing the axis and which direction is positive.

\[
d = y + u
\]

\[
\text{Vol. slice} = \pi x^2 \, dy = \pi (y^2 - y^2) \, dy
\]

\[
F = \rho g \text{ } V = \rho g \pi (y^2 - y^2) \, dy
\]

\[
W = \int_{5}^{7} \rho g \pi (y^2 - y^2) (y + u) \, dy
\]

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**DO NOT WRITE IN THIS TABLE.**

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Cross section of the

\[ x^2 + y^2 = 7^2 \]
Alternate method.

Slice

\[ V = \pi x^2 \Delta y \]
\[ = \pi (y_1 - (y_1 - y)^2) \Delta y \]

The eqn of the circle when not centered at the origin.

\[ (x-h)^2 + (y-k)^2 = r^2 \]
\[ x^2 + (y-7)^2 = 49 \]

\[ W = \int_0^2 \rho g \pi (y_1 - (y_1 - y)^2) \cdot (11-y) \, dy \]