MATH 152, Spring 2019
COMMON EXAM II - VERSION A

LAST NAME(print): _________________________ FIRST NAME(print): _________________________

INSTRUCTOR: _________________________

SECTION NUMBER: ______________

DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1 (Problems 1-18), mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2 (Problems 19-23), present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: _________________________

Some integrals that may or may not be useful.

\[
\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C
\]

\[
\int \csc^3 x \, dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln |\csc x - \cot x| + C
\]
PART I: Multiple Choice. 3.5 points each

1. Which of the following is the form of the partial-fraction decomposition for the rational function \( \frac{7}{(x + 3)^2(x^2 - 9)(x^2 + 16)} \)

(a) None of these.
(b) \( \frac{A}{x + 3} + \frac{B}{(x + 3)^2} + \frac{C}{(x + 3)^3} + \frac{D}{x - 3} + \frac{E}{x + 4} + \frac{F}{x - 4} \)
(c) \( \frac{Ax^2 + Bx + C}{(x + 3)^3} + \frac{D}{x - 3} + \frac{Ex + F}{x^2 + 16} \)
(d) \( \frac{A}{x + 3} + \frac{B}{(x + 3)^2} + \frac{C}{(x + 3)^3} + \frac{D}{x - 3} + \frac{Ex + F}{x^2 + 16} \)
(e) \( \frac{A}{x + 3} + \frac{B}{(x + 3)^2} + \frac{Cx}{(x^2 - 9)} + \frac{Ex + F}{x^2 + 16} \)

2. Which of the following integrals is/are improper?

I. \( \int_0^2 \frac{1}{x - 3} dx \)  
II. \( \int_{-\infty}^0 \frac{1}{x^2 + 1} dx \)  
III. \( \int_2^5 x^2 \ln(5 - x) dx \)

(a) II only  
(b) I and II  
(c) I, II, and III  
(d) III only  
(e) II and III

3. After an appropriate substitution, the integral \( \int \frac{\sqrt{9x^2 - 4}}{x^2} dx \) is equivalent to which of the following?

(a) \( \int \frac{3\tan^2 \theta}{\sec \theta} d\theta \)  
(b) \( \int \frac{3\cos^2 \theta}{\sin^2 \theta} d\theta \)  
(c) \( \int \frac{3\sec^3 \theta}{\tan^2 \theta} d\theta \)  
(d) \( \int \frac{9\tan \theta}{2\sec^2 \theta} d\theta \)  
(e) \( \int \frac{9\sec \theta}{2\tan^2 \theta} d\theta \)
4. Compute \( \int_{1}^{3} \frac{3x^2 + 2x + 3}{x^2 + 1} \, dx \)

(a) \( 6 + \ln(10) + \ln(2) \)
(b) \( 6 + 2 \arctan(3) + 2 \arctan(1) \)
(c) \( 6 + \ln(10) - \ln(2) \)
(d) \( 6 + 2 \arctan(3) - 2 \arctan(1) \)
(e) \( 6 - \ln(10) + \ln(2) \)

5. Which of these series diverge by the Test for Divergence?

(a) \( \sum_{n=1}^{\infty} \sin \left( \frac{1}{n} \right) \)
(b) \( \sum_{n=1}^{\infty} \ln \left( \frac{n}{n + 7} \right) \)
(c) None of these.
(d) \( \sum_{n=1}^{\infty} \ln \frac{n}{n} \)
(e) \( \sum_{n=1}^{\infty} \frac{1}{5 - e^{-n}} \)

6. Compute the sum of the series \( \sum_{n=1}^{\infty} \frac{4^{n+1}}{5^n} \).

(a) This series diverges.
(b) 16
(c) 12
(d) None of these.
(e) 20
7. Decide on the convergence/divergence of each of the following improper integrals.

(I) \( \int_{3}^{\infty} \frac{x - 2}{x^4} \, dx \)  
(II) \( \int_{1}^{\infty} \frac{1}{\sqrt[3]{x}} \, dx \)

(a) More information is needed to make a decision.
(b) (I) is convergent and (II) is convergent
(c) (I) is convergent and (II) is divergent
(d) (I) is divergent and (II) is divergent
(e) (I) is divergent and (II) is convergent

8. Which of the following is the appropriate trigonometric substitution for \( \int \frac{1}{\sqrt{x^2 + 8x + 25}} \, dx \)?

(a) \( x + 4 = 5 \sec \theta \)
(b) None of these.
(c) \( x + 4 = 9 \tan \theta \)
(d) \( x^2 + 8x = 5 \tan \theta \)
(e) \( x + 4 = 3 \tan \theta \)

9. The series \( \sum_{n=1}^{\infty} \left( e^{1/n} - e^{1/(n+1)} \right) \)

(a) converges to 0
(b) converges to \( e - 1 \)
(c) None of these.
(d) diverges
(e) converges to \( e \)
10. The integral $\int_{-2}^{0} \frac{2}{x^3} \, dx$

(a) None of these.
(b) diverges to $\infty$.
(c) converges to $\frac{1}{4}$
(d) diverges to $-\infty$.
(e) converges to $-\frac{1}{4}$

11. The integral $\int_{3}^{\infty} \frac{5 + 2 \sin x}{x} \, dx$ is

(a) divergent by comparison with $\int_{3}^{\infty} \frac{3}{x} \, dx$
(b) divergent by comparison with $\int_{3}^{\infty} \frac{7}{x} \, dx$
(c) convergent by comparison with $\int_{3}^{\infty} \frac{3}{x} \, dx$
(d) divergent by comparison with $\int_{3}^{\infty} \frac{5}{x} \, dx$
(e) convergent by comparison with $\int_{3}^{\infty} \frac{7}{x} \, dx$

12. Assume that the sequence $\{a_n\}$ is decreasing and bounded below by 1, i.e. $a_n \geq 1$, for all positive $n$. Determine if the sequence is convergent or divergent.

$a_1 = 4$ and $a_{n+1} = \frac{10}{7 - a_n}$

(a) Divergent
(b) Convergent to 1
(c) Convergent to 2
(d) Convergent to $\frac{10}{7}$
(e) Convergent to 5
13. The sequence \( a_n = \frac{(-1)^n n^2}{2n^2 + 5} \)

(a) Converges to \( \frac{1}{2} \)
(b) Converges to \( -\frac{1}{2} \)
(c) None of these.
(d) Diverges
(e) Converges to 0

14. Compute \( \int 2\sqrt{1-x^2} \, dx \)

(a) \( \arcsin(x) + x\sqrt{1-x^2} + C \)
(b) \( \arcsin(x) - x\sqrt{1-x^2} + C \)
(c) \( \ln |x + \sqrt{x^2 - 1}| - x\sqrt{x^2 - 1} + C \)
(d) \( x\sqrt{1+x^2} + \ln |x + \sqrt{1+x^2}| + C \)
(e) \( x\sqrt{1-x^2} - \ln |x + \sqrt{1-x^2}| + C \)

15. Let \( \sum_{n=1}^{\infty} a_n \) be a series whose \( n \)th partial sum is \( s_n = \frac{n}{n+2} \). Find \( a_4 \).

(a) \( \frac{2}{3} \)
(b) None of these.
(c) \( \frac{1}{21} \)
(d) 1
(e) \( \frac{1}{15} \)
16. Which of the following sequences is both bounded and decreasing?

(a) \( a_n = 1 - \frac{1}{n^2} \)
(b) \( a_n = \left(\frac{\pi}{4}\right)^n \)
(c) None of these.
(d) \( a_n = \left(\frac{-1}{2}\right)^n \)
(e) \( a_n = \ln(n + 5) - \ln(n^2 + 1) \)

17. The sequence \( a_n = \frac{n^2}{n - 1} - \frac{n^2}{n + 4} \)

(a) Converges to 0
(b) None of these.
(c) Converges to 3
(d) Converges to 5
(e) Diverges

18. Using the Remainder Estimate for the Integral Test, find an upper bound on the error in using \( s_4 \) to approximate \( \sum_{n=1}^{\infty} \frac{3}{(n+1)^2} \).

(a) \( \frac{1}{12} \)
(b) \( \frac{3}{25} \)
(c) \( \frac{3}{4} \)
(d) \( \frac{6}{125} \)
(e) \( \frac{3}{5} \)
PART II WORK OUT

Directions: Present your solutions in the space provided. Show all your work neatly and concisely and Box your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

19. (4 points) The series $\sum_{n=1}^{\infty} a_n$ has partial sums given by $s_n = \frac{2n^2 + 3}{4n^2 + 5}$. Determine if the series converges or diverges. If it converges, give the sum.

20. (6 points) Find a general formula, $a_n$, for the sequence. Assume the pattern continues, and begins with $n = 1$.

\[ \left\{ \frac{1}{5}, -\frac{4}{8}, \frac{9}{11}, -\frac{16}{14}, \frac{25}{17}, \cdots \right\} \]
21. (9 points) Compute \( \int \frac{\sqrt{16 + x^2}}{x^4} \, dx \).
22. (9 points) Determine whether the series below converges or diverges. Fully support your answer.

\[ \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2} \]
23. (9 points) Compute \( \int \frac{7x^2 + 3x + 11}{(x + 1)(x^2 + 4)} \, dx \)