DIRECTIONS:

1. The use of a calculator, laptop or computer is prohibited.

2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

3. In Part 1, mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore for your own records, also record your choices on your exam!

4. In Part 2, present your solutions in the space provided. Show all your work neatly and concisely and clearly indicate your final answer. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

5. Be sure to fill in your name, UIN, section number and version letter of the exam on the ScanTron form.

THE AGGIE CODE OF HONOR

“An Aggie does not lie, cheat or steal, or tolerate those who do.”

Signature: ________________________________
1. Which of the following is equal to $\int_{0}^{1} x^2 \sin(x - 1) dx$?

A. $\int_{-1}^{0} (u + 1)^2 \sin(u) du$.
B. $\int_{0}^{1} (u + 1)^2 \sin(u) du$.
C. $\int_{-1}^{0} u^2 \sin(u) du$.
D. $\int_{0}^{1} x^2 \sin(u) du$.
E. $\int_{0}^{1} u^2 \sin(u) du$.

2. Find the area bounded by the curves $y = 2 - x^2$ and $y = x^2$.

A. $\frac{4}{3}$.
B. 1.
C. $-\frac{8}{3}$.
D. $\frac{8}{3}$.
E. $\frac{22}{12}$. 
3. Compute \( \int_1^e \ln(x^2) \, dx \).
   
   A. \( \frac{1-e}{e} \).
   
   B. \( 2e - 1 \).
   
   C. \( 2 \).
   
   D. \( 4e - 2 \).
   
   E. 1.

4. Which integral gives the volume of the solid obtained by rotating the region bounded by \( y = x^3 \), \( y = 0 \), and \( x = 1 \) about the line \( x = 2 \)?
   
   A. \( 2\pi \int_0^2 x^3(1+x) \, dx \).
   
   B. \( 2\pi \int_0^1 x^3(1+x) \, dx \).
   
   C. \( 2\pi \int_0^2 x^3(2-x) \, dx \).
   
   D. \( 2\pi \int_0^1 x^3(2-x) \, dx \).
   
   E. \( \pi \int_0^1 (2 - y^{\frac{1}{3}})^2 \, dy \).

5. Compute \( \int_0^{\frac{\pi}{2}} \cos^2(\theta) \sin^3(\theta) \, d\theta \).
   
   A. \( -\frac{2}{15} \).
   
   B. \( \frac{2}{15} \).
   
   C. \( -\frac{\pi^2}{30} \).
   
   D. 0.
   
   E. \( \frac{\pi^2}{30} \).
6. Which of the follow integrals gives the area of the region bounded by the curves \( y = \frac{1}{x} \), \( y = \frac{x - 1}{e(e - 1)} \) and \( x = 1 \). (Hint: the curves \( y = \frac{1}{x} \) and \( y = \frac{x - 1}{e(e - 1)} \) intersect at the point \((e, \frac{1}{e})\)).

A. \( \int_e^1 \left( \frac{1}{x} - \frac{x - 1}{e(e - 1)} \right) dx \).

B. \( \int_1^e \frac{dy}{y} + \int_0^1 (e(e - 1)y + 1) dy \).

C. \( \int_0^e \left( \frac{1}{x} - \frac{x - 1}{e(e - 1)} \right) dx \).

D. \( \int_1^e \left( \frac{x - 1}{e(e - 1)} - \frac{1}{x} \right) dx \).

E. \( \int_0^1 (\frac{1}{y} - 1) dy \).

7. Find the volume of the solid obtained by rotating the region bounded by \( y = x^2 \) and \( y = x^3 \) (in the first quadrant) around the \( x \)-axis.

A. \( \frac{\pi}{35} \).

B. \( \frac{1}{12} \).

C. \( \frac{\pi}{12} \).

D. \( \frac{1}{105} \).

E. \( \frac{2}{35} \).
8. A 90ft cable weighing 10 lb is hanging down the side of a 200 ft building. There is a 50 lb bucket of rocks attached to the rope. How much work is required to pull the rope with the bucket of rocks 30 feet up the side of the building?
   A. 6000 ft-lb.
   B. 1750 ft-lb.
   C. 1500 ft-lb.
   D. 1800 ft-lb.
   E. 1550 ft-lb.

9. Which of the following is equal to \( \int_{0}^{\pi/4} \tan^2(\theta) \sec^4(\theta) \, d\theta \)?
   A. \( \int_{0}^{\pi/4} u^2(1 - u^2) \, du \).
   B. \( \int_{0}^{\pi/4} u^2(1 + u^2) \, du \).
   C. \(- \int_{0}^{1} u^2(1 + u^2) \, du \).
   D. \( \int_{0}^{1} u^2(1 - u^2) \, du \).
   E. \( \int_{0}^{1} u^2(1 + u^2) \, du \).

10. Compute \( \int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \).
    A. \(-6\).
    B. \(-2e^2 + 2e\).
    C. \(2e^2 - 2e\).
    D. \(6\).
    E. \(2e - 2\).
11. Compute \(\int_{0}^{\sqrt{\pi}} x \sin(\pi - x^2) \, dx\).

A. \(-\sin \sqrt{\pi}\)
B. \(-1\)
C. 2
D. \(-2\)
E. 1

12. Compute \(\int_{0}^{1} x e^{2x} \, dx\).

A. \(\frac{e^2 + 1}{4}\)
B. \(\frac{3e^2 + 1}{4}\)
C. \(6e^2\)
D. \(-2e^2\)
E. \(\frac{e + 1}{4}\)

13. An ideal spring (i.e. it obeys Hooke’s Law) has a natural length of 10 meters. The work done in stretching the spring from 14 meters to 18 meters is 24J. Determine the spring constant.

A. \(k = \frac{1}{2} \frac{N}{m}\)
B. \(k = 3 \frac{N}{m}\)
C. \(k = \frac{3}{8} \frac{N}{m}\)
D. \(k = \frac{1}{m}\)
E. \(k = 3 \frac{N}{14 m}\).
14. Find the area between \( y = \cos x \) and \( y = \sin x \) from \( x = 0 \) to \( x = \frac{5\pi}{4} \).

A. \( \sqrt{2} - 1 \)
B. \((\sqrt{2} - 1) - 2\sqrt{2}\)
C. \( 1 - 3\sqrt{2} \)
D. \( 1 + \sqrt{2} \).
E. \( 3\sqrt{2} - 1 \)

15. Which integral gives the volume of the solid obtained by rotating the region bounded by \( y = \sin x \) and \( y = \frac{2}{\pi} x \) \textit{in the first quadrant} around the \( y \)-axis? (It might be of use to know that these curves intersect when \( x = -\frac{\pi}{2}, x = 0, \) and \( x = \frac{\pi}{2} \)).

A. \( 2\pi \int_{\frac{\pi}{2}}^{\pi} \left(\sin x - \frac{2}{\pi} x\right)dx \)
B. \( 2\pi \int_{0}^{\frac{\pi}{2}} x\left(\sin x - \frac{2}{\pi} x\right)dx \)
C. \( \pi \int_{0}^{\frac{\pi}{2}} x\left(\sin x - \frac{2}{\pi} x\right)dx \)
D. \( 2\pi \int_{\frac{\pi}{2}}^{\pi} x\left(\sin x - \frac{2}{\pi} x\right)dx \)
E. \( \pi \int_{0}^{\frac{\pi}{2}} \left(\sin^2 x - \frac{4}{\pi^2} x^2\right)dx \).
16. Which of the following is equal to \( \int_{0}^{\pi} x^2 \cos x \, dx \)? (Hint: just do integration by parts once.)

A. \( \pi - 2 \int_{0}^{\pi} x \sin x \, dx \).

B. \( -2 \int_{0}^{\pi} x \sin x \, dx \).

C. \( -\pi + 2 \int_{0}^{\pi} x \sin x \, dx \).

D. \( \pi + 2 \int_{0}^{\pi} x \sin x \, dx \).

E. \( 2 \int_{0}^{\pi} x \sin x \, dx \).

\textbf{PART II WORK OUT}

\textbf{Directions:} Present your solutions in the space provided. \textit{Show all your work} neatly and concisely and \textit{Box your final answer}. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (9 points) Compute \( \int \cos^2 x \sin^2 x \, dx \).
18. (9 points) Compute \( \int e^{2x} \sin x \, dx \).
19. (9 points) Find the volume of the solid whose base is the circle $x^2 + y^2 = 1$ and whose cross sections perpendicular to the $y$ axis are equilateral triangles. (You may find it helpful to know that the area of an equilateral triangle is $\frac{\sqrt{3}}{4} \ell^2$ where $\ell$ is the sidelenath.)
20. (9 points) A hemispherical tank of radius 2 ft is filled with a foot of a liquid with weight density $\rho g \frac{\text{lb}}{\text{ft}^3}$. There is a one foot spout mounted at the top of the tank through which water is drained. Set up the integral that gives the work required to pump the water out of the tank through the spout. **Indicate on the picture where you are placing the axes and which direction is positive.** DO NOT INTEGRATE.
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