Instructions
Once you start the exam, you must finish. The timer will not stop if you take a break.
You may use your notes, calculator, and book. Remember that Webassign is stupid. It
can not understand what you meant to type. Be careful with how you enter your
answers. Good Luck.

1. Question Details
SCalcET8 7.3.002. [3805202]
Evaluate the integral using the indicated trigonometric substitution. (Use C for the constant of integration.)
\[
\int \frac{x^3}{\sqrt{x^2 + 100}} \, dx, \quad x = 10 \tan(\theta)
\]

\[C + \frac{1}{3} (x^2 + 100)^{3/2} - 100\sqrt{x^2 + 100}\]

Sketch and label the associated right triangle.

Solution or Explanation
Click to View Solution
Evaluate the integral.
\[
\int_{1/2}^{1} \frac{x}{\sqrt{x^2 - 7}} \, dx
\]

Solution or Explanation
Click to View Solution

Evaluate the integral.
\[
\int_{2/4}^{1/2} \frac{dx}{x^5 \sqrt{16x^2 - 1}}
\]

Solution or Explanation
Click to View Solution
Evaluate the integral.
\[
\int_{0}^{1} \frac{12}{4x^2 + 5x + 1} \, dx
\]

Solution or Explanation
\[
\frac{12}{4x^2 + 5x + 1} = \frac{A}{4x + 1} + \frac{B}{x + 1}
\]
Multiply both sides by \((4x + 1)(x + 1)\) to get \(12 = A(x + 1) + B(4x + 1)\). The coefficients of \(x\) must be equal and the constant terms are also equal, so \(A + 4B = 0\) and \(A + B = 12\). Subtracting the second equation from the first gives \(B = -4\), and hence, \(A = 16\). Thus,
\[
\int_{0}^{1} \frac{12}{4x^2 + 5x + 1} \, dx = \int_{0}^{1} \left( \frac{16}{4x + 1} - \frac{4}{x + 1} \right) \, dx = \left[ 4 \ln|4x + 1| - 4 \ln|x + 1| \right]_{0}^{1} = (4 \ln(5) - 4 \ln(2)) - 0 = 4 \ln \left( \frac{5}{2} \right).
\]

Another method: Substituting \(-1\) for \(x\) in the equation \(12 = A(x + 1) + B(4x + 1)\) gives \(12 = -3B \iff B = -4\). Substituting \(\frac{1}{4}\) for \(x\) gives \(12 = \frac{3}{4}A \iff A = 16\).

Evaluate the integral.
\[
\int_{0}^{1} \frac{x - 4}{x^2 - 5x + 6} \, dx
\]

Solution or Explanation
\[
\frac{x - 4}{x^2 - 5x + 6} = \frac{A}{x - 2} + \frac{B}{x - 3}
\]
Multiplying both sides by \((x - 2)(x - 3)\) to get \(x - 4 = A(x - 3) + B(x - 2) \iff x - 4 = Ax - 3A + Bx - 2B \iff x - 4 = (A + B)x + (-3A - 2B)\).
The coefficients of \(x\) must be equal and the constant terms are also equal, so \(A + B = 1\) and \(-3A - 2B = -4\). Adding twice the first equation to the second gives us \(-A = -2 \iff A = 2\), and hence, \(B = -1\). Thus,
\[
\int_{0}^{1} \frac{x - 4}{x^2 - 5x + 6} \, dx = \int_{0}^{1} \left( \frac{2}{x - 2} - \frac{1}{x - 3} \right) \, dx = \left[ 2 \ln|x - 2| - \ln|x - 3| \right]_{0}^{1} = (0 - \ln 2) - (2 \ln 2 - \ln 3) = \ln(3) - 3 \ln(2).
\]

Another Method: Substituting \(3\) for \(x\) in the equation \(x - 4 = A(x - 3) + B(x - 2)\) gives \(-1 = B\). Substituting \(2\) for \(x\) gives \(-2 = -A \iff A = 2\).
Write out the form of the partial fraction decomposition of the function (as in this example). Do not determine the numerical values of the coefficients.

\[
\frac{x^4 - 2x^3 + x^2 + 2x - 1}{x^4 - 2x + 1}
\]

(a) \[\frac{A}{x - 1} + \frac{B}{(x - 1)^2} + x^2\]

(b) \[\frac{A}{x} + \frac{Bx + C}{x^2 + x + 1}\]

Solution or Explanation

(a) \[\frac{x^4 - 2x^3 + x^2 + 2x - 1}{x^4 - 2x + 1} = \frac{x^2(x^2 - 2x + 1) + 2x - 1}{x^2 - 2x + 1} = x^2 + \frac{2x - 1}{(x - 1)^2}. \text{ [or use long division]}\]

(b) \[\frac{x^2 - 1}{x^3 + x^2 + x} = \frac{x^2 - 1}{x(x^2 + x + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + x + 1}\]

Determine whether the integral is convergent or divergent.

\[\int_{9}^{\infty} \frac{1}{(x - 8)^{3/2}} \, dx\]

- convergent
- divergent

If it is convergent, evaluate it. (If the quantity diverges, enter DIVERGES.)

\[2\]

Solution or Explanation

\[\int_{9}^{\infty} \frac{1}{(x - 8)^{3/2}} \, dx = \lim_{t \to \infty} \int_{9}^{t} (x - 8)^{-3/2} \, dx = \lim_{t \to \infty} \left[ -2(x - 8)^{-1/2} \right]_{9}^{t} \quad [u = x - 8, \, du = dx] \]

\[= \lim_{t \to \infty} \left( -\frac{2}{\sqrt{t - 8}} + \frac{2}{\sqrt{1}} \right) = 0 + 2 = 2. \text{ Convergent} \]
9. Question Details

Determine whether the integral is convergent or divergent.

\[ \int_{0}^{\infty} \frac{x^2}{\sqrt{5 + x^3}} \, dx \]

- convergent
- divergent

If it is convergent, evaluate it. (If the quantity diverges, enter DIVERGES.)

\[ \text{DIVERGES} \]

Solution or Explanation

\[ \int_{0}^{\infty} \frac{x^2}{\sqrt{5 + x^3}} \, dx = \lim_{t \to \infty} \int_{0}^{t} \frac{x^2}{\sqrt{5 + x^3}} \, dx = \lim_{t \to \infty} \left[ \frac{2}{3} \sqrt{5 + x^3} \right]_{0}^{t} = \lim_{t \to \infty} \left( \frac{2}{3} \sqrt{5 + t^3} - \frac{2}{3} \right) = \infty \]

Divergent

10. Question Details

Determine whether the integral is convergent or divergent.

\[ \int_{-\infty}^{0} ze^{3z} \, dz \]

- convergent
- divergent

If it is convergent, evaluate it. (If the quantity diverges, enter DIVERGES.)

\[ -\frac{1}{9} \]

Solution or Explanation

\[ \int_{-\infty}^{0} ze^{3z} \, dz = \lim_{t \to -\infty} \int_{t}^{0} ze^{3z} \, dz = \lim_{t \to -\infty} \left[ \frac{1}{3}ze^{3z} - \frac{1}{9}e^{3z} \right]_{t}^{0} \]

integration by parts with \( u = z \), \( dv = e^{3z} \, dz \)

\[ = \lim_{t \to -\infty} \left[ \left( 0 - \frac{1}{9} \right) - \left( \frac{1}{3}e^{0} - \frac{1}{9}e^{0} \right) \right] = -\frac{1}{9} - 0 + 0 \] [by l'Hospital's Rule] = \(-\frac{1}{9}\) Convergent.
11. Question Details
Determine whether the integral is convergent or divergent.

\[ \int_{-2}^{14} \frac{13}{\sqrt{x + 2}} \, dx \]

- [ ] convergent
- [ ] divergent

If it is convergent, evaluate it. (If the quantity diverges, enter DIVERGES.)

\[ \frac{116}{3} \]

Solution or Explanation
Click to View Solution

12. Question Details
Use the Comparison Theorem to determine whether the integral is convergent or divergent.

\[ \int_{0}^{\pi} \frac{38}{\sqrt{x}} \sin^2(x) \, dx \]

- [ ] convergent
- [ ] divergent

Solution or Explanation
Click to View Solution

13. Question Details
Determine whether the sequence converges or diverges. If it converges, find the limit. (If an answer does not exist, enter DNE.)

\[ a_n = \frac{4 + 8n^2}{n + 8n^2} \]

\[ \lim_{n \to \infty} a_n = \boxed{1} \]

Solution or Explanation

\[ a_n = \frac{4 + 8n^2}{n + 8n^2} = \frac{(4 + 8n^2)\sin^2(x)}{(n + 8n^2)\sin^2(x)} \]

\[ = \frac{4 + 4n^2}{n + 8n^2} \]

so

\[ a_n \to \frac{8 + 0}{8 + 0} = 1 \quad \text{as} \quad n \to \infty. \]

Converges
14. Question Details

Use a graph of the sequence to decide whether the sequence is convergent or divergent. If the sequence is convergent, guess the value of the limit from the graph and then prove your guess. (If an answer does not exist, enter DNE.)

\[ a_n = 1 + \left(-\frac{2}{e}\right)^n \]

\[ \lim_{n \to \infty} a_n = \boxed{1} \]

Solution or Explanation

From the graph, it appears that the sequence converges to 1. \((-\frac{2}{e})^n\) converges to 0, and hence \(1 + (-\frac{2}{e})^n\) converges to \(1 + 0 = 1\).

15. Question Details

Determine whether the sequence converges or diverges. If it converges, find the limit. (If an answer does not exist, enter DNE.)

\[ a_n = \ln(n + 5) - \ln(n) \]

\[ \lim_{n \to \infty} a_n = \boxed{0} \]

Solution or Explanation

\[ a_n = \ln(n + 5) - \ln(n) = \ln\left(\frac{n + 5}{n}\right) = \ln\left(1 + \frac{5}{n}\right) = \ln(1) = 0 \text{ as } n \to \infty \] because \(\ln\) is continuous. Converges

16. Question Details

Calculate the sum of the series \(\sum_{n=1}^{\infty} a_n\) whose partial sums are given.

\[ s_n = 3 - 9(0.8)^n \]

\[ \boxed{3} \]

Solution or Explanation

\[ \sum_{n=1}^{\infty} a_n = \lim_{n \to \infty} s_n = 3 - 9 \lim_{n \to \infty} (0.8)^n = 3 - 9(0) = 3 \]
17. Let \( a_n = \frac{5n}{7n + 1} \)

(a) Determine whether \( \{a_n\} \) is convergent.

- convergent
- divergent

(b) Determine whether \( \sum_{n=1}^{\infty} a_n \) is convergent.

- convergent
- divergent

18. Determine whether the series is convergent or divergent by expressing \( s_n \) as a telescoping sum (as in Example 8).

\[
\sum_{n=4}^{\infty} \frac{6}{n^3 - 1}
\]

- convergent
- divergent

If it is convergent, find its sum. (If the quantity diverges, enter DIVERGES.)

\[
\frac{7}{4}
\]
19. Question Details

Find the values of $x$ for which the series converges. (Enter your answer using interval notation.)

$$
\sum_{n=1}^{\infty} (-5)^n x^n
$$

Find the sum of the series for those values of $x$.

Solution or Explanation

$$
\sum_{n=1}^{\infty} (-5)^n x^n = \frac{-5x}{1+5x}
$$

20. Question Details

(a) What is the difference between a sequence and a series?

- A series is an ordered list of numbers whereas a sequence is the sum of a list of numbers.
- A sequence is an ordered list of numbers whereas a series is an unordered list of numbers.
- A sequence is an ordered list of numbers whereas a series is the sum of a list of numbers.
- A sequence is an unordered list of numbers whereas a sequence is the sum of a list of numbers.
- A sequence is an unordered list of numbers whereas a series is the sum of a list of numbers.

(b) What is a convergent series? What is a divergent series?

- A convergent series is a series for which $\lim_{n \to \infty} a_n$ exists. A series is convergent if it is not divergent.
- A series is divergent if the $n$th term converges to zero. A series is convergent if it is not divergent.
- A series is divergent if the sequence of partial sums is a convergent sequence. A series is convergent if it is not divergent.
- A series is convergent if the $n$th term converges to zero. A series is divergent if it is not convergent.
- A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.

Solution or Explanation

(a) A sequence is an ordered list of numbers whereas a series is the sum of a list of numbers.
(b) A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.
21. Question Details

Determine whether the series is convergent or divergent.

\[ \sum_{n=2}^{\infty} \frac{2}{n \ln(n)} \]

- convergent
- divergent

Solution or Explanation
Click to View Solution

22. Question Details

Consider the following function.

\[ f(x) = \frac{9 \cos(\pi x)}{\sqrt{x}} \]

What conclusions can be made about the series \( \sum_{n=1}^{\infty} \frac{9 \cos(\pi n)}{\sqrt{n}} \) and the Integral Test?

- The Integral Test can be used to determine whether the series is convergent since the function is positive and decreasing on \([1, \infty)\).
- The Integral Test can be used to determine whether the series is convergent since the function is not positive and not decreasing on \([1, \infty)\).
- The Integral Test can be used to determine whether the series is convergent since it does not matter if the function is positive or decreasing on \([1, \infty)\).
- The Integral Test cannot be used to determine whether the series is convergent since the function is not positive and not decreasing on \([1, \infty)\).
- There is not enough information to determine whether or not the Integral Test can be used or not.

Solution or Explanation
The function \( f(x) = \frac{9 \cos(\pi x)}{\sqrt{x}} \) is neither positive nor decreasing on \([1, \infty)\), so the hypotheses of the Integral Test are not satisfied for the series \( \sum_{n=1}^{\infty} \frac{9 \cos(\pi n)}{\sqrt{n}} \).