MATH 152, Fall 2020
COMMON EXAM III - VERSION A

LAST NAME(print): ____________________________________________ FIRST NAME(print): ________________________________

INSTRUCTOR: ____________________________________________

SECTION NUMBER: ________________________________________

DIRECTIONS:

1. The use of a calculator, notes, or non-approved webpages is prohibited.

2. Each question has a pdf version. Click on that link and a pdf version of that question will open in a new window. Note: For multiple choice questions, the answer choices are in no particular order. You will need to select the corresponding answer choice in eCampus.

3. For workout questions, work these problems on the answer template that was provided by your instructor. If your instructor did not provide the template, use your own paper. Don’t forget to write your Name and UIN at the top of the page for every work out question.

4. When you are done with the exam, you will use your phone to scan the solutions to the workout questions into a single pdf file and submit this pdf file to Gradescrope. Only submit the solutions to the workout questions. Do not include any solutions to the multiple choice.

5. Show all your work neatly and concisely. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.
PART I: Multiple Choice. 4 points each

1. What statement below is true regarding the series \( \sum_{n=3}^{\infty} \frac{5 + 2 \cos n}{\sqrt{n}} \)?

(a) Divergent by the Comparison Test with \( \sum_{n=3}^{\infty} \frac{3}{\sqrt{n}} \)

(b) Convergent by the Comparison Test with \( \sum_{n=3}^{\infty} \frac{3}{\sqrt{n}} \)

(c) Convergent by the Comparison Test with \( \sum_{n=3}^{\infty} \frac{7}{\sqrt{n}} \)

(d) Divergent by the Comparison Test with \( \sum_{n=3}^{\infty} \frac{7}{\sqrt{n}} \)

(e) Divergent by the Comparison Test with \( \sum_{n=3}^{\infty} \frac{5}{\sqrt{n}} \)

\[ \frac{3}{\sqrt{n}} \leq \frac{5 + 2\cos n}{\sqrt{n}} \leq \frac{7}{\sqrt{n}} \]

2. Which of the following three tests will establish that the series \( \sum_{n=3}^{\infty} \frac{1}{n^2 - 7} \) will converge?

(\(\checkmark\)) (I) The Limit Comparison Test with \( \sum_{n=3}^{\infty} \frac{1}{n^2} \)

(\(\times\)) (II) The Comparison Test with \( \sum_{n=3}^{\infty} \frac{1}{n^2} \)

(\(\times\)) (III) The Alternating Series Test.

(a) I only
(b) I and II only
(c) II only
(d) I and III only
(e) II and III only

3. Let \( s = \sum_{n=1}^{\infty} \frac{(-2)^n}{n+2} \). Using the Alternating Series Estimation Theorem, which of the following is a true statement regarding \( |R_4| = |s - s_4| \)?

(\(\checkmark\)) (a) \( |R_4| = |s - s_4| \leq \frac{2^5}{7!} \)

(b) \( |R_4| = |s - s_4| \leq \frac{2^5}{7!} \)

(c) \( |R_4| = |s - s_4| \leq \frac{2^4}{6!} \)

(d) \( |R_4| = |s - s_4| \leq \frac{2^4}{6!} \)

(e) \( |R_4| = |s - s_4| \leq \frac{2^6}{8!} \)

\[ |R_4| \leq b_5 = \frac{2^5}{7!} \]
4. For which series is the Ratio Test inconclusive?

(a) \[ \sum_{n=1}^{\infty} \frac{\ln(n)}{n^5} \]
(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \]
(c) \[ \sum_{n=1}^{\infty} \frac{n^3}{3^n} \]
(d) \[ \sum_{n=1}^{\infty} \frac{2^n}{n^{2n} + n} \]
(e) \[ \sum_{n=1}^{\infty} \frac{4^{n+5}}{(n+1)!} \]

5. Which of the following statements is true for this pair of series.

(I) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \]  
(II) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \]

- (a) (I) is conditionally convergent, (II) is absolutely convergent.
- (b) Both series are absolutely convergent.
- (c) Both series are conditionally convergent.
- (d) (I) is absolutely convergent, (II) is conditionally convergent.
- (e) None of these are true.

6. Compute the limit, \( L \), used by the Ratio Test to determine the convergence/divergence of this series. \[ \sum_{n=1}^{\infty} \frac{n!n^3}{(2n)!} \]

\( L = \frac{3}{4} \)
(b) \( L = 0 \)
(c) \( L = \infty \)
(d) \( L = \frac{3}{2} \)
(e) \( L = 3 \)

7. Find the radius of convergence of the series \[ \sum_{n=1}^{\infty} \frac{(n+1)! (x+3)^n}{5^n} \]

- (a) 0
- (b) \( \infty \)
- (c) 1
- (d) 5
- (e) \( \frac{1}{5} \)

\[ \lim_{n \to \infty} \left| \frac{(n+2)! (x+3)^{n+1}}{5^{n+1} (n+1)! (x+3)^n} \cdot \frac{5^n}{(x+3)} \right| = \infty \]
8. Which of the following is a power series centered at 0 for the function \( f(x) = \frac{1}{9 - 4x^2} \)?

(a) \( \sum_{n=0}^{\infty} \frac{4^n x^{2n}}{9^{n+1}} \)

(b) \( \sum_{n=0}^{\infty} \frac{9^n x^{2n}}{4^{n+1}} \)

(c) \( \sum_{n=0}^{\infty} (-1)^n \frac{4^n x^{2n}}{9^{n+1}} \)

(d) \( \sum_{n=0}^{\infty} (-1)^n \frac{9^n x^{2n}}{4^{n+1}} \)

(e) None of these.

9. Suppose that the series \( \sum_{n=1}^{\infty} c_n x^n \) converges at \( x = -4 \) and diverges at \( x = 6 \). Which of the following statements is true?

(i) \( \sum_{n=1}^{\infty} c_n 4^n \) converges.

(\( \times \))

(ii) \( \sum_{n=1}^{\infty} c_n 7^n \) diverges.

(\( \checkmark \))

(iii) \( \sum_{n=1}^{\infty} c_n 5^n \) may or may not converge.

(a) I and II only

(b) II only

(c) I and II only

(d) I, II, and III

(e) II only

10. Find the radius of convergence for the Maclaurin series representation of \( f(x) = \frac{2x}{5 - 3x} \).

(a) \( R = \frac{5}{3} \)

(b) \( R = 5 \)

(c) \( R = \frac{3}{5} \)

(d) \( R = 1 \)

(e) None of these.
11. Using a known Maclaurin series for \( \sin x \), express \( \int \frac{\sin x}{x} \, dx \) as a power series.

(a) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!(2n+1)} \]

(b) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+2}}{(2n+1)(2n+2)} \]

(c) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)^2} \]

(d) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n+1} \]

(e) \[ C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!(2n+1)} \]

12. Compute the sum of the infinite series \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n)!} \).

(a) \(-\pi\)

(b) \(-1\)

(c) 1

(d) 0

(e) \(\pi\)

13. Find the 15th derivative at \( x = 3 \), i.e. \( f^{(15)}(3) \), for \( f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-3)^n}{(n+5)7^n} \).

(a) \( f^{(15)}(3) = \frac{15!}{20(7^{15})} \)

(b) \( f^{(15)}(3) = \frac{-15!}{20(7^{15})} \)

(c) \( f^{(15)}(3) = \frac{1}{20(7^{15})} \)

(d) \( f^{(15)}(3) = \frac{-1}{20(7^{15})} \)

(e) None of these.

14. Which of the following is the Maclaurin series for \( f(x) = e^{-x^2} \)?

(a) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \]

(b) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2n!} \]

(c) \[ \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \]

(d) \[ \sum_{n=0}^{\infty} \frac{x^{2n}}{n!} \]

(e) \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \]
15. Find the Taylor polynomial $T_4(x)$, the 4th degree Taylor polynomial, for the function $f(x) = \frac{1}{1 - 5x^2}$ centered at $a = 0$.

(a) $T_4(x) = 1 - 5x^2 + 25x^4$
(b) $T_4(x) = 1 + 5x^2 + 25x^4$
(c) $T_4(x) = 1 - 5x^2 + 25x^4 - 125x^6$
(d) $T_4(x) = 1 + 5x^2 + 25x^4 + 125x^6$
(e) None of these.

16. Find the coefficient of the $(x - 1)^3$ term for the Taylor series centered at $x = 1$ for the function

$$f(x) = x^3 + 3x^4 + 2x^2 + 6x + 7.$$

(a) 22
(b) 32
(c) $\frac{60x^2 + 72x}{6}$
(d) 44
(e) $60x^2 + 72x$

**PART II WORK OUT**

**Directions:** Present your solutions in the space provided. *Show all your work neatly and concisely and Box your final answer.* You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

17. (8 points) Find the Maclaurin series representation for the function $f(x)$. You are only allowed to use shortcuts provided by your instructor.

$$f(x) = \frac{x}{(1 - 7x)^2}$$

Let

$$g(x) = \frac{1}{1 - 7x} = \sum_{n=0}^{\infty} 7^n x^n$$

Then

$$g' = \frac{7}{(1 - 7x)^2} = \sum_{n=1}^{\infty} 7^n x^{n-1}$$

Hence

$$f(x) = \frac{x}{7} g'(x) = \frac{x}{7} \sum_{n=1}^{\infty} 7^n x^{n-1} = \sum_{n=1}^{\infty} 7^{n-1} x^n$$
18. (9 points) Evaluate the indefinite integral as a Maclaurin series.

\[ \int x \arctan(4x^2) \, dx \]

\[ = \int x \sum_{n=0}^{\infty} (-1)^n \frac{(4x^2)^{2n+1}}{2n+1} \, dx \]

\[ = \int x \sum_{n=0}^{\infty} (-1)^n \frac{4^{2n+1}}{2n+1} \cdot x \cdot \frac{4^{n+2}}{2n+1} \, dx \]

\[ = \int \sum_{n=0}^{\infty} (-1)^n \frac{4^{2n+1}}{2n+1} \cdot x \cdot \frac{4^{n+3}}{2n+1} \, dx \]

\[ = C + \sum_{n=0}^{\infty} (-1)^n \frac{4^{2n+1}}{2n+1} \cdot \frac{4^{n+4}}{(4n+4)(2n+1)} \]
19. (9 points) Find the Taylor series for the function \( f(x) = \frac{1}{(x+1)^2} \) centered at \( a = 2 \). Express your answer in summation notation.

\[
f(2) = (x+1)^{-2}
\]

\[
f'(x) = -2(x+1)^{-3}
\]

\[
f''(x) = 2 \cdot 3 \cdot (x+1)^{-4}
\]

\[
f'''(x) = -2 \cdot 3 \cdot 4 \cdot (x+1)^{-5}
\]

\[
f^{(n)}(x) = (-1)^n \frac{(n+1)!}{(x+1)^{n+2}}
\]

\[
c_n = \frac{f^{(n)}(2)}{n!} = \frac{1}{n!} \cdot \frac{(-1)^n (n+1)!}{3^{n+2}}
\]

\[
c_n = \frac{(-1)^n (n+1)}{3^{n+2}}
\]

\[
f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)}{3^{n+2}} \cdot (x-2)^n
\]
20. (10 points) Find the radius of convergence and the interval of convergence of the power series. You must test your endpoints for convergence.

\[ \sum_{n=1}^{\infty} \frac{3^n(x-1)^n}{\sqrt{n}} \]

\[ \lim_{n \to \infty} \left| \frac{\frac{3^{n+1}}{\sqrt{n+1}}}{\frac{3^n}{\sqrt{n}}}(x-1)^n \right| = \lim_{n \to \infty} \left| \frac{3}{1} \cdot \frac{\sqrt{n}}{\sqrt{n+1}} \right| \]

\[ = \left| 3(x-1) \right| \]

\[ |3(x-1)| < 1 \]
\[ |x-1| < \frac{1}{3} \]
\[ R = \frac{1}{3} \]

\[ -\frac{1}{3} < x-1 < \frac{1}{3} \]
\[ -\frac{2}{3} < x < \frac{4}{3} \]

\[ x = \frac{4}{3} \]  \[ \sum_{n=1}^{\infty} \frac{3^n \left( \frac{4}{3} - 1 \right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n \left( \frac{1}{3} \right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \text{ d.n. } p \text{-series } p = \frac{1}{2} \]

\[ x = \frac{2}{3} \]  \[ \sum_{n=1}^{\infty} \frac{3^n \left( \frac{2}{3} - 1 \right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n \left( \frac{1}{3} \right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \text{ converge by A.S.T.} \]

I: \( \left[ \frac{2}{3}, \frac{4}{3} \right) \)