1. Question Details

Suppose \( \sum a_n \) and \( \sum b_n \) are series with positive terms and \( \sum b_n \) is known to be convergent.

(a) If \( a_n > b_n \) for all \( n \), what can you say about \( \sum a_n \)? Why?
- \( \sum a_n \) diverges by the Comparison Test.
- We cannot say anything about \( \sum a_n \).
- \( \sum a_n \) converges by the Comparison Test.

(b) If \( a_n < b_n \) for all \( n \), what can you say about \( \sum a_n \)? Why?
- \( \sum a_n \) diverges by the Comparison Test.
- We cannot say anything about \( \sum a_n \).
- \( \sum a_n \) converges by the Comparison Test.

Solution or Explanation

(a) We cannot say anything about \( \sum a_n \). If \( a_n > b_n \) for all \( n \) and \( \sum b_n \) is convergent, then \( \sum a_n \) could be convergent or divergent. (See this note.)

(b) If \( a_n < b_n \) for all \( n \), then \( \sum a_n \) is convergent. [This is part (i) of the Comparison Test.]
2. **Question Details**

Determine whether the series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{3 + \sin n}{n^{3/2}} \]

- diverges by Comparison with \[ \sum_{n=1}^{\infty} \frac{4}{n^{3/2}} \]
- diverges by Comparison with \[ \sum_{n=1}^{\infty} \frac{2}{n^{3/2}} \]
- Diverges by the Test for Divergence
- diverges by Comparison with \[ \sum_{n=1}^{\infty} \frac{2}{n^{3/2}} \]
- diverges by Comparison with \[ \sum_{n=1}^{\infty} \frac{3}{n^{3/2}} \]
- Converges by the Test for Divergence
- converges by Comparison with \[ \sum_{n=1}^{\infty} \frac{3}{n^{3/2}} \]
- converges by Comparison with \[ \sum_{n=1}^{\infty} \frac{4}{n^{3/2}} \]

3. **Question Details**

Determine whether the series converges or diverges.

\[ \sum_{n=3}^{\infty} \frac{n^3}{n^2 - 3} \]

- Diverges by the Comparison test with \[ \sum_{n=3}^{\infty} \frac{1}{n^2} \]
- converges by the Limit Comparison test with \[ \sum_{n=3}^{\infty} \frac{1}{n^2} \]
- Converges by the Comparison test with \[ \sum_{n=3}^{\infty} \frac{1}{n^2} \]
- Diverges by the Test for Divergence
- Converges by the Test for Divergence
- Diverges by the Ratio Test
- Diverges by the Limit Comparison test with \[ \sum_{n=3}^{\infty} \frac{1}{n^2} \]
- Converges by the Ratio Test
4. **Question Details**

The series, \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \), is convergent by the Alternating Series Test. Use the Alternating Series Estimation Theorem to determine the minimum number of terms needed to approximate the sum of the series with an error less than \( \frac{1}{100} \).

\[ \text{minimum number of terms needed} = 4 \] terms

5. **Question Details**

Which of the following is true regarding the series \( \sum_{n=0}^{\infty} \frac{5n^2 2^{2n}}{8^n} \)?

- The Ratio test limit is \( \frac{15}{8} \), so the series diverges.
- The Ratio test limit is \( \frac{9}{8} \), so the series converges.
- The Ratio test limit is \( \frac{15}{8} \), so the series converges.
- The Ratio test limit is \( \frac{9}{8} \), so the series diverges.
- The Ratio test limit is \( \frac{3}{8} \), so the series converges.
- The Ratio test limit is \( \frac{45}{8} \), so the series diverges.
- The Ratio test limit is \( \frac{45}{8} \), so the series converges.
- The Ratio test limit is \( \frac{3}{8} \), so the series diverges.

6. **Question Details**

Which of the following statements is true regarding the convergence of the series below?

\[ (I) \sum_{n=0}^{\infty} \frac{(-1)^n n}{n^2 + 5} \quad (II) \sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} \]

- Both I and II are conditionally convergent
- I is absolutely convergent; II is divergent
- I is divergent; II is conditionally convergent
- Both I and II are absolutely convergent
- I is absolutely convergent; II is conditionally convergent
- I is conditionally convergent; II is absolutely convergent
- Both I and II are divergent
- I is divergent; II is absolutely convergent
7. Question Details

The series \( \sum_{n=0}^{\infty} c_n x^n \) converges when \( x = -4 \) and diverges when \( x = 8 \). Which of the following series is guaranteed to converge?

(I) \( \sum_{n=0}^{\infty} c_n 2^n \)

(II) \( \sum_{n=0}^{\infty} c_n 4^n \)

(III) \( \sum_{n=0}^{\infty} c_n (-8)^n \)

- I and II only
- I and III only
- I, II, and III
- II only
- II and III only
- I only
- None are guaranteed to converge.
- III only

8. Question Details

Find the radius of convergence, \( R \), of the series.

\[ \sum_{n=0}^{\infty} \frac{(-1)^n (x - 6)^n}{2n + 1} \]

\( R = \boxed{1} \)

Find the interval, \( I \), of convergence of the series. (Enter your answer using interval notation.)

\( I = \boxed{(5, 7]} \)

Solution or Explanation

Click to View Solution

9. Question Details

Find the radius of convergence, \( R \), of the series.

\[ \sum_{n=0}^{\infty} \frac{x^n + 4}{2n!} \]

\( R = \boxed{\infty} \)
10. Question Details  
Find the radius of convergence, $R$, of the following series.  
\[ \sum_{n=1}^{\infty} n!(2x-1)^n \]  
\[ R = \boxed{0} \]

11. Question Details  
Find a power series representation for the function. (Give your power series representation centered at $x = 0$.)  
\[ f(x) = \frac{x^2}{x^4 + 81} \]  
\[ f(x) = \sum_{n=0}^{\infty} \left( -\frac{1}{3} \right)^n \frac{x^{4n+2}}{3^{2n+4}} \]  
Determine the interval of convergence. (Enter your answer using interval notation.)  
-3, 3

Solution or Explanation  
\[ f(x) = \frac{x^2}{x^4 + 81} = \frac{x^2}{81} \frac{1}{1 + \left(\frac{x}{3}\right)^4} = \frac{x^2}{81} \sum_{n=0}^{\infty} \left( -\frac{x}{3} \right)^{4n} \]  
or, equivalently, \[ \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{3^{2n+4}} \]  
The series converges when \[ \left| -\frac{x}{3} \right|^4 < 1 \Rightarrow \left| \frac{x}{3} \right| < 1 \Rightarrow \left| x \right| < 3 \], so $R = 3$ and $I = (-3, 3)$.

12. Question Details  
Find a power series for $f(x)$. (Give your power series representation centered at $x = 0$.)  
\[ f(x) = \frac{x}{1 + 4x^2} \]  
\[ f(x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{4^n x^{2n-1}}{2n} \]

13. Question Details  
Find a power series representation for the function. (Give your power series representation centered at $x = 0$.)  
\[ f(x) = x^9 \tan^{-1}(x^2) \]  
\[ f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+11}}{2n+1} \]

Solution or Explanation  
\[ f(x) = x^9 \tan^{-1}(x^2) = x^9 \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \]  
[by this example] \[ = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{2n+1} + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+11}}{2n+1} \]  
for $|x^2| < 1 \Rightarrow |x| < 1$, so $R = 1$. 

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14. Question Details

Find the Maclaurin series for the given function.

\[ f(x) = x^3 \cos(6x) \]

\[
\sum_{n=0}^{\infty} \left( \frac{(-1)^n 6^n x^{2n+3}}{(2n)!} \right)
\]

15. Question Details

Find the Taylor series for \( f \) centered at 6 if

\[ f^{(n)}(6) = \frac{(-1)^n}{4^n(n+2)} \]

\[
\sum_{n=0}^{\infty} \left( \frac{(-1)^n}{n+2} \right)
\]

Solution or Explanation

Click to View Solution

16. Question Details

Find the sum of the series.

\[
\sum_{n=0}^{\infty} \frac{2^{n+1}}{9^n n!}
\]

\[ \frac{2e^{2/9}}{9} \]

17. Question Details

Find the Taylor series for \( f(x) \) centered at the given value of \( a \).

\[ f(x) = \frac{7}{x^2}, \quad a = 5 \]

\[
\sum_{n=0}^{\infty} \left( \frac{7(-1)^n 5^{-n-2}(n+1)(x-5)^n}{n!} \right)
\]

18. Question Details

Consider the following function. \( f(x) = \ln(3 + x), \quad a = 2, \quad n = 3 \)

Approximate \( f(x) \) by a Taylor polynomial with degree \( n \) at the number \( a \).

\[
T_3(x) = \frac{1}{3!} (x-2)^3 + \frac{1}{5!} (x-2)^2 + \frac{x-2}{5} + \ln(5)
\]