MATH 253.504–506
Final Exam, version A
12/13/99

An unsupported answer is a wrong answer!

This test has 2 pages, 13 problems, and 175 points.

Conversion from spherical to Cartesian coordinates:

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \\ dV &= \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi \end{aligned}$$

1. (10 pts.) Find the equation of the plane tangent to the surface $xy^2z^3 = 2$ at the point $(2, 1, 1)$.

2. (10 pts.) Evaluate the line integral $\int_{\mathcal{C}} y\,dx - x\,dy$ on the semicircle $x^2 + y^2 = 4$, $y \geq 0$, from the point $(2, 0)$ to $(-2, 0)$.

3. (15 pts.) Reverse the order of integration to write $\displaystyle\int_1^3 \int_{6/x}^{-2x+8} \phi(x, y)\,dy\,dx$ as an iterated integral in the order $dx\,dy$. (You won't be evaluating an integral in this problem.)

4. (10 pts.) Write a parameterization for the part of the cylinder $x^2 + z^2 = 4$, which lies between the planes $y = -1$ and $x + 2y + z = 8$. Be sure to specify the parameter domain.

5. (15 pts.) Using the divergence theorem, compute $\iint_{\mathcal{S}} (\vec{F} \cdot \vec{n})\,dS$, where $\mathcal{S}$ is the surface of the cylinder $x^2 + y^2 \leq 1$, $0 \leq z \leq 2$, $\vec{n}$ is the outward pointing unit normal, and $\vec{F} = \langle xy^2, xz, x^2z \rangle$.

6. (20 pts.) Suppose that $\mathcal{E}$ is the region in space bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 4$. Write (**but don't evaluate**) $\iiint_{\mathcal{E}} x^2\,dV$ in
   (a) spherical coordinates.
   (b) cylindrical coordinates.

7. (15 pts.) Set up (**but do not evaluate**) an iterated integral, in the order $dz\,dy\,dx$, for $\iiint_{\mathcal{E}} z\,dV$, where $\mathcal{E}$ is the region in space bounded below by $z = x^2 + y^2$ and above by the plane $2x + 4y - z = -4$.

8. (15 pts.) Set up (**but do not evaluate**) an iterated integral in u and v to find the flux of $\vec{F} = \langle 2x, -z, y \rangle$ across the surface $\vec{r}(u, v) = \langle u^2, uv, v^2 \rangle$, $1 \leq u \leq 2$, $1 \leq v \leq 3$, using the upward pointing unit normal.

9. Suppose that $z = f(x, y)$, where $x = 5u + 2v$, $y = 3u + v$, and that f has continuous second partials.

   (a) (5 pts.) Find $\dfrac{\partial z}{\partial u}$ in terms of u, v, and the partial derivatives of f.

   (b) (10 pts.) Find $\dfrac{\partial^2 z}{\partial u \partial v}$ in terms of u, v, and the partial derivatives of f.

10. (15 pts.) Find $\oint_{\mathcal{C}} x^3\, dx + (x^3 + y^2)\, dy$, where $\mathcal{C}$ consists of the line segment from $(-1, 0)$ to $(1, 0)$ followed by the parabolic arc $y = 1 - x^2$ from $(1, 0)$ to $(-1, 0)$ (see diagram).

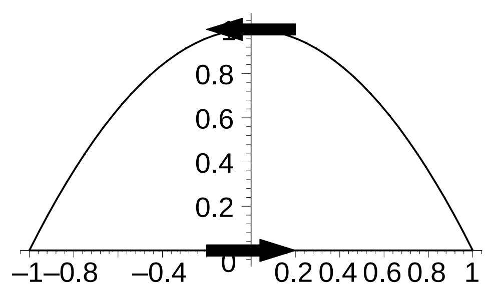

Figure for problem 10.

11. (10 pts.) Determine the equation of the plane which contains the point $(1, -1, 2)$ and the line $x = -1 + 2t$, $y = 1 + t$, $z = 1 + 3t$.

12. (10 pts.) Describe the domain and range of the function $f(x, y) = \sqrt{16 - 4x^2} - \sqrt{9 - y^2}$.

13. (15 pts.) Determine the maximum and minimum values of $f(x, y) = x^2 + 2y^2 - x$ on the disk $x^2 + y^2 \leq 1$, giving the points where they occur.